2011_mdtrm_b_soln

Thursday, March 10, 2011

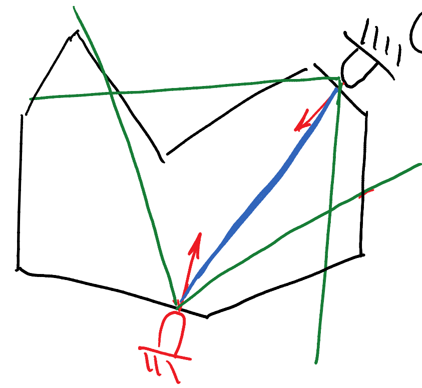
9:00 PM

20 pts

1.) A planar object is grasped with two hard fingers. The coefficient of friction at both contact points is 1.0.

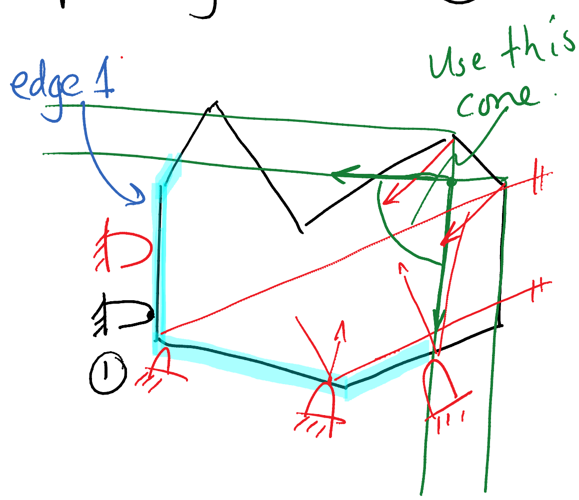
12pts

a.) Find a location for contact ② such that a 2-fingered grasp has frictional form closure.



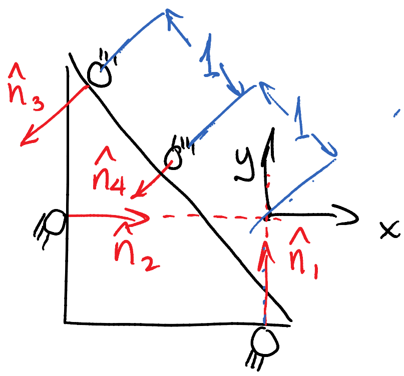
8pts

b.) You know contact ① is somewhere on edge 1, but its precise location is not known. Find a finite region on the polygon such that placing contact ② anywhere in that region, will form a grasp with frictional form closure.



10 points

2.) Show analytically that the grasp shown on the right does not have form closure.



Show that the following implication does not hold:

$$G_n^T \nu \geq 0 \Rightarrow \nu = 0$$

$$G_n^T = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ -0.8 & -0.6 & 2 \\ -0.8 & -0.6 & 1 \end{bmatrix}$$

$$\nu = \begin{bmatrix} v_x \\ v_y \\ \omega_z \end{bmatrix}$$

$$G_n^T \nu = \quad v_y \geq 0 \qquad (1)$$

$$v_x \geq 0 \qquad (2)$$

$$-0.8\, v_x - 0.6\, v_y + 2\omega_z \geq 0 \qquad (3)$$

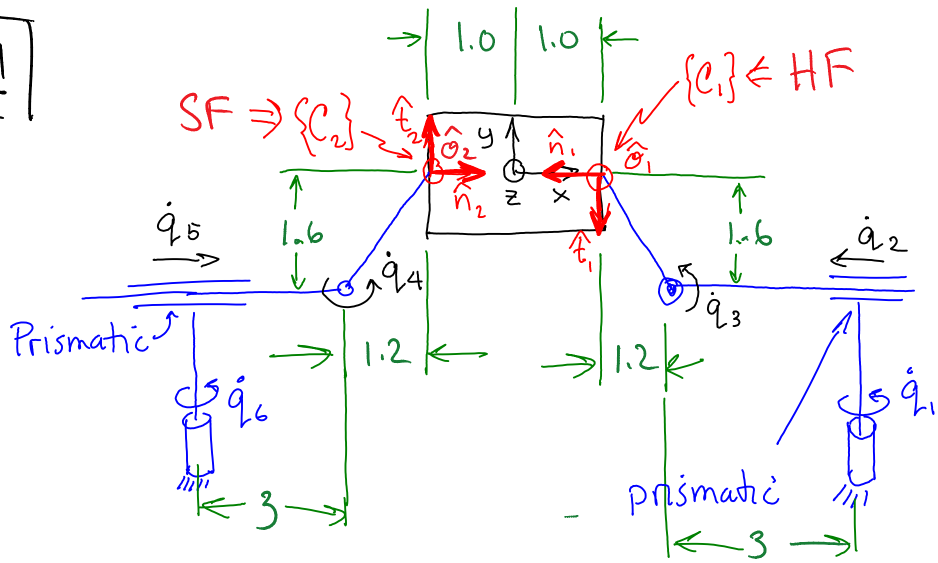
$$-0.8\, v_x - 0.6\, v_y + \omega_z \geq 0 \qquad (4)$$

Pick any positive values of v_x & v_y. Then $.8 v_x + .6 v_y$ will be strictly positive. Any ω_z greater than $.8 v_x + .6 v_y$ satisfies inequalities (3) & (4)

q.e.d.

30pts

3.) 3D Problem!



Contact 1 (on right) is a hard finger contact.

Contact 2 (on left) is a soft finger contact.

(10pts)

a.) Construct G & J using the (x-y-z) reference frame shown.

$$
G = \begin{array}{c}
\left[\begin{array}{ccc|cccc}
-1 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & -1 & 0 & 0 & 1 & 0 \\
0 & -1 & 0 & 0 & -1 & 0 & 0
\end{array}\right] \\
\begin{array}{ccc|cccc}
f_{1n} & f_{1t} & f_{1o} & f_{2n} & f_{2t} & f_{2o} & m_{2n}
\end{array}
\end{array}
$$

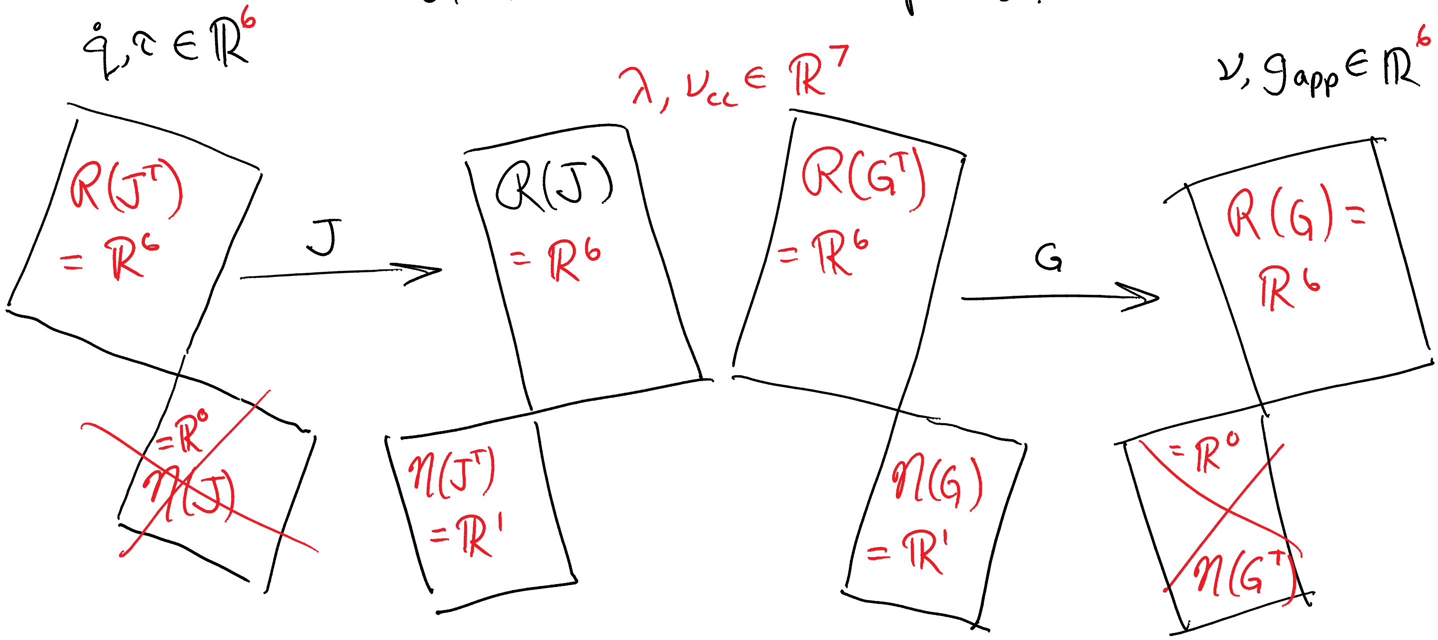
$$
J = \begin{array}{cc}
\left[\begin{array}{ccc|ccc}
0 & 1 & 1.6 & & & \\
0 & 0 & 1.2 & & 0 & \\
4.2 & 0 & 0 & & & \\
\hline
 & & & -1.6 & 1 & 0 \\
 & 0 & & 1.2 & 0 & 0 \\
 & & & 0 & 0 & -4.2 \\
 & & & 0 & 0 & 0
\end{array}\right] &
\begin{array}{c}
f_{1n} \\ f_{1t} \\ f_{1o} \\ f_{2n} \\ f_{2t} \\ f_{2o} \\ m_{2n}
\end{array}
\end{array}
$$

(10pts)

b.) For the correct G and J, bases of the four null spaces are:

$$N(G) = \begin{bmatrix} -1 \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad N(G^T) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad N(J) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad N(J^T) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Complete the picture below, i.e. identify the dimensions of the various subspaces.



For the next two problems you might find the following quantities helpful.

$$J^T G^+ = \begin{bmatrix} 0 & 0 & 2.1 & 0 & -2.1 & 0 \\ -0.5 & 0 & 0 & 0 & 0 & 0 \\ -0.8 & -0.6 & 0 & 0 & 0 & -0.6 \\ -0.8 & 0.6 & 0 & 0 & 0 & -0.6 \\ 0.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2.1 & 0 & -2.1 & 0 \end{bmatrix} \qquad G\,N(J^T) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$G(J^T)^+ = \begin{bmatrix} 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 4/3 & -5/6 & 5/6 & 4/3 & 0 \\ 5/21 & 0 & 0 & 0 & 0 & -5/21 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -5/21 & 0 & 0 & 0 & 0 & -5/21 \\ 0 & 4/3 & -5/6 & -5/6 & -4/3 & 0 \end{bmatrix} \qquad J^T N(G) = \begin{bmatrix} 0 \\ -\sqrt{2}/2 \\ -1.131 \\ 1.131 \\ -\sqrt{2}/2 \\ 0 \end{bmatrix}$$

5pts c.) Use the relationships $\tau = J^T\lambda$ and $g_{app} = -G\lambda$ to determine which joint torques do not change in response to changes in the internal wrench.

$\lambda = -G^+ g_{app} + \underbrace{N(G)\alpha}_{\text{Internal wrench}}$ where α is an arbitrary scalar.

$\tau = J^T\lambda$. τ induced by the internal wrench is:

$\tau = J^T \lambda$. τ induced by the internal wrench is:

$$J^T N(G)\alpha = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix} \Rightarrow \tau_1 \text{ \& } \tau_6 \text{ are unaffected.}$$

(5pts)

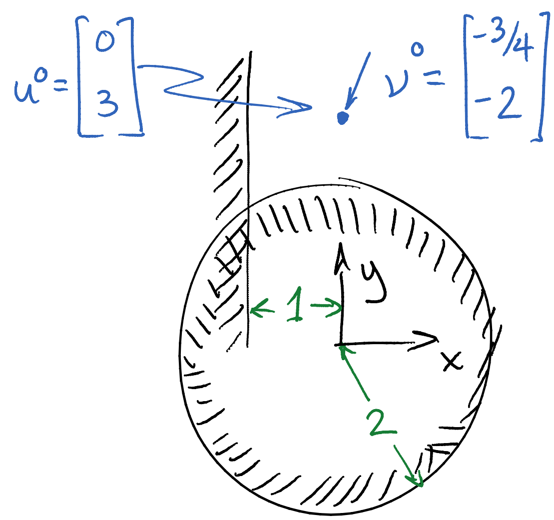
d.) Use the relationships $\tau = J^T \lambda$ and $g_{app} = -G\lambda$ to determine which component of the external wrench cannot be controlled by adjusting joint torques.

$g_{app} = -G(J^T)^+ \tau - G N(J^T)\beta$ where β is an arbitrary scalar.

Notice that the 4th row of $G(J^T)^+$ is zero. Therefore $g_{app}(4)$ (i.e., moments about the x-axis cannot be controlled by adjusting τ.

(40 pts)

4.) A particle is close to circular and linear obstacles ($x^2 + y^2 \geq R^2$ and $x \geq -1$).



(30pts)

a.) Assume $\mu = 0$, $m = h = 1$.

Determing u, ν, and p_n at $t=1$ and $t=2$.

$$M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad G_n^0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \Psi_n^0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

The discrete Newton-Euler equation becomes for time step 1

$$\nu^1 = p_n^1 + \nu^0 \qquad (1)$$

Normal complementarity is:

$$0 \le p_n^{l+1} \perp G_n^T \nu^{l+1} + \Psi_n^l \ge 0$$

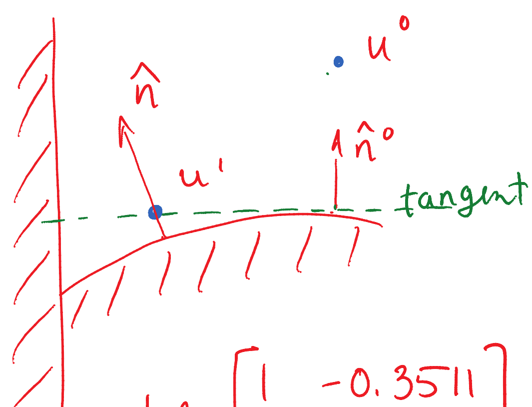
substituting (1), the LCP for time step 1 is:

$$0 \le p_n^1 \perp p_n^1 + \nu^0 + \Psi_n^0 \ge 0$$

$$0 \le p_n^1 \perp p_n^1 + \begin{bmatrix} 1/4 \\ -1 \end{bmatrix} \ge 0$$

$$\Rightarrow p_n^1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Rightarrow \nu^1 = \begin{bmatrix} -3/4 \\ -2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -3/4 \\ -1 \end{bmatrix}$$

$$u^1 = \begin{bmatrix} 0 \\ 3 \end{bmatrix} + \begin{bmatrix} -3/4 \\ -1 \end{bmatrix} = \begin{bmatrix} -3/4 \\ 2 \end{bmatrix}$$



$$G_n^1 \simeq \begin{bmatrix} 1 & -0.3511 \\ 0 & 0.9363 \end{bmatrix}$$

For the second time step

G_n & Ψ_n change.

<u>NE</u>:

$$\nu^2 = G_n p_n^2 + \nu^1$$

Normal Complementarity:

$$0 \le p_n^2 \perp G_n^T \nu^2 + \psi_n^1 \ge 0$$

$$0 \le p_n^2 \perp G_n^T (G_n p_n^2 + \nu^1) + \psi_n^1 \ge 0$$

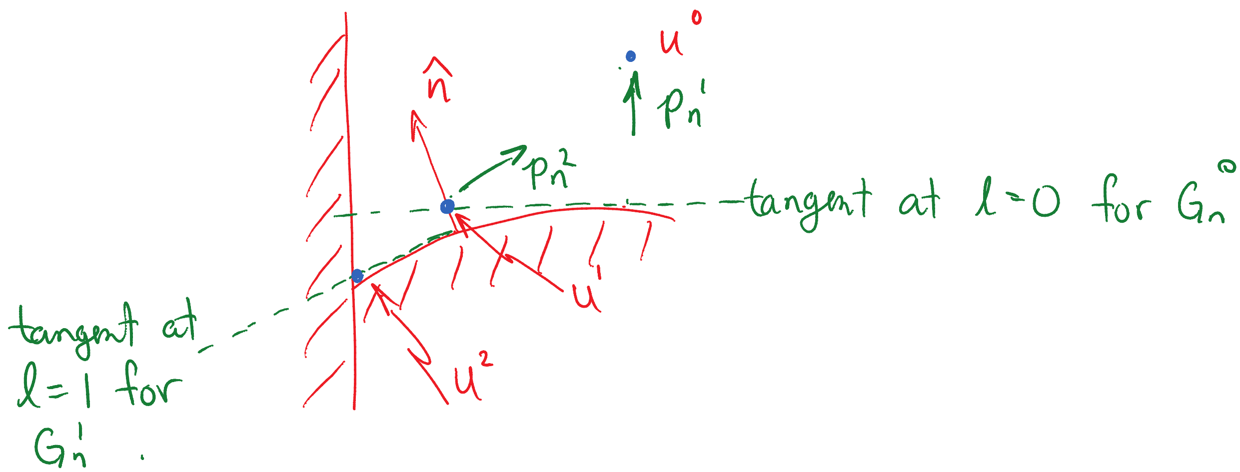
$$\psi_n^1 \simeq \begin{bmatrix} 1/4 \\ 0.1 \end{bmatrix} \qquad G_n^T G_n \cong \begin{bmatrix} 1 & 0.4 \\ 0.4 & 1 \end{bmatrix}$$

Four possible cases, but we know from the physical situation that p_{1n} & p_{2n} should be positive. ∴ Solve

$$G_n^T G_n p_n^2 = -G_n^T \nu^1 - \psi_n^1$$

$$p_n^2 = -(G_n^T G_n)^{-1} G_n^T \nu^1 - (G_n^T G_n)^{-1} \psi_n^1$$

$$p_n^2 = \begin{bmatrix} 0.7854 \\ 0.8128 \end{bmatrix} \qquad \nu^2 = \begin{bmatrix} -0.25 \\ -0.239 \end{bmatrix} \qquad u^2 = \begin{bmatrix} -1.000 \\ 1.761 \end{bmatrix}$$



(5pts) b.) Assuming $\mu_1, \mu_2 \neq 0$, give the definitions of G_n, G_f, E, U, M, and $\frac{\partial \psi_n}{\partial t}$, for the first time step.

$$G_n^0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad G_f = \begin{bmatrix} 0 & 0 & -1 & 1 \\ 1 & -1 & 0 & 0 \end{bmatrix} \quad E = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}^T$$

$$U = \begin{bmatrix} \mu_1 & 0 \\ 0 & \mu_2 \end{bmatrix} \quad M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \frac{\partial \psi_n}{\partial t} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(5pts) c.) What is the size of the LCP if both obstacles are incorporated and friction is not zero?

The dimension of the big matrix is the size of the LCP.

$$\begin{bmatrix} M & -G_n & -G_f & 0 \\ G_n^T & 0 & 0 & E \\ G_f^T & 0 & 0 & 0 \\ 0 & U & -E^T & 0 \end{bmatrix}$$

(10×10)

It is 10.