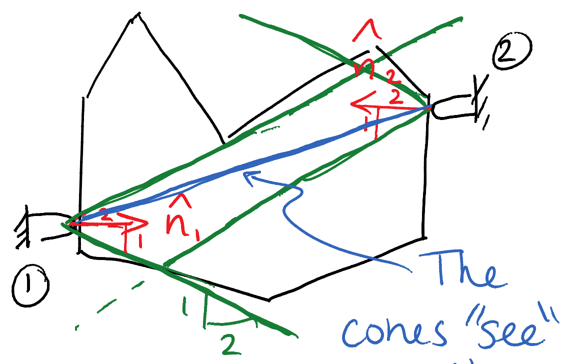
2011_mdtrm_soln

Tuesday, March 01, 2011
2:43 PM

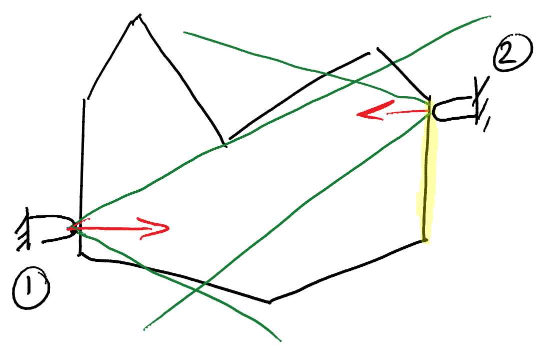
1. A planar object is grasped with two hard fingers. The coefficient of friction at both contact points is 0.5.

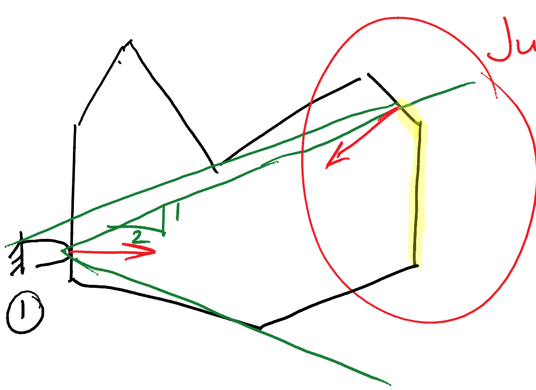
a. Show graphically that the grasp to the right has frictional form closure



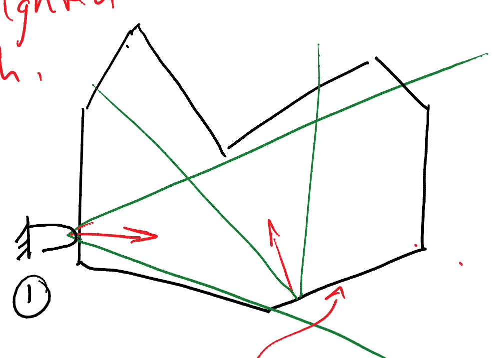
The cones "see" each other q.e.d.

b. Assume contact ① is fixed. Highlight the edges of the object where the second contact could be placed such the grasp has frictional form closure.





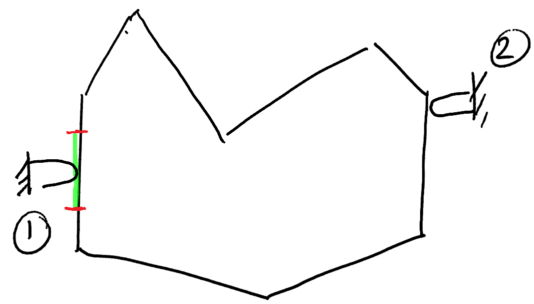
Just this highlighted patch.



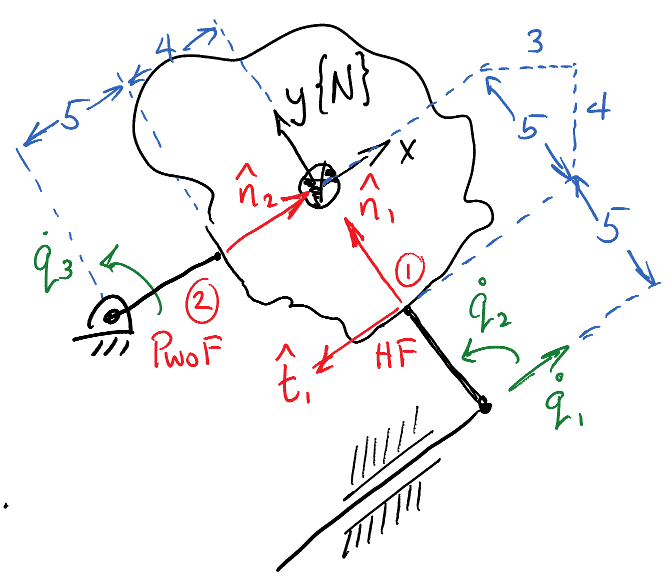
No closure points on this edge.

c. Suppose the robot is

not perfectly accurate, so when attempting to place finger 1 as shown, one can only guarantee that it is placed in the contact region shown. Assume also that finger 2 is perfectly accurate. Sketch the region of finger 2 placements that would yield frictional form closure for every placement of finger 1 inside the region shown.



.2 The hand in the planar system to the right makes two contacts with the object. Contact 1 is modeled as a hard finger (point w/ friction) contact.



the other as a point w/o friction.

a. Determine G & J

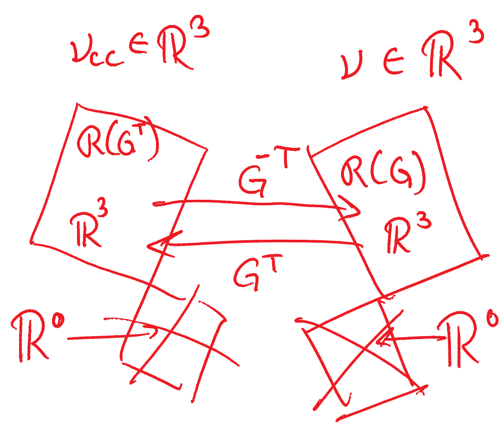
$$J = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 5 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad G^T = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & -5 \\ 1 & 0 & 0 \end{bmatrix}$$

If you use {N} with origin at center of gravity and axes oriented like so, then

$$G^T = \begin{bmatrix} 0.6 & -0.8 & 0.8 \\ 0.8 & -0.6 & 0.6 \\ 0 & 0 & -5 \end{bmatrix}$$

b. If the contact points on the fingers could move arbitrarily, could they be chosen to cause any desired $\nu \in \mathbb{R}^3$?

Yes, because Rank(G) = 3, G provides a "1-to-1" and "onto" mapping between ν_{cc} & ν, i.e. for every $\nu \in \mathbb{R}^3$ $\exists$ exactly one $\nu_{cc} \ni G^{-T}\nu_{cc} = \nu$.



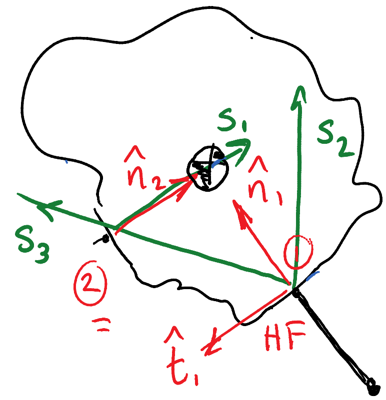
c. Does this grasp have form closure?

No. There must be at least 4 contacts for form cl. in the plane.

d. Does the grasp have force closure.

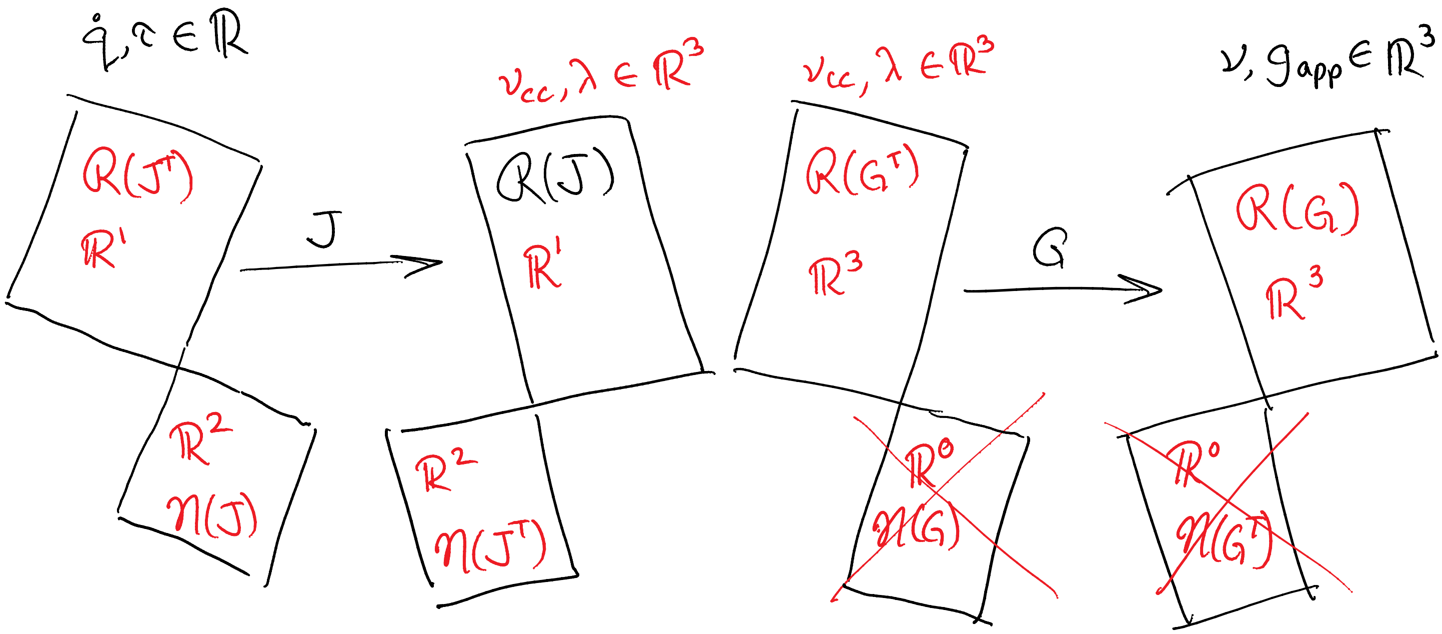
No. $\mathrm{Rank}(G)=3$, $\mathrm{Rank}(GJ)=1$.

Another necessary condition is at least 4 distinct friction cone edges.



The grasp has only 3: s_1, s_2, s_3.

e. Complete the picture below, i.e. identify the dimensions of the various subspaces.



$\text{Rank}(J) = 1 \Rightarrow \text{Dim}(\mathcal{R}(J)) = 1 = \text{Dim}(\mathcal{R}(J^T))$

f. Identify an element of $\mathcal{R}(J)$ and interpret it in physical terms specifically applied to the hand. You can explain in terms of velocities or forces, which ever you are more comfortable with.

$$\mathcal{R}(J) = \text{column space of } J = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \alpha_1 + \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix} \alpha_2 + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \alpha_3$$

arbitrary scalars

The elements of $\mathcal{R}(J)$ are: $J\dot{q} = \begin{bmatrix} v_{1n} \\ v_{1t} \\ v_{2n} \end{bmatrix} = \begin{bmatrix} 0 \\ -\alpha_1 + 5\alpha_2 \\ 0 \end{bmatrix}$

$\therefore$ each element of $\mathcal{R}(J)$ has $v_{1n} = v_{2n} = 0$, v_{1t} arbitrary

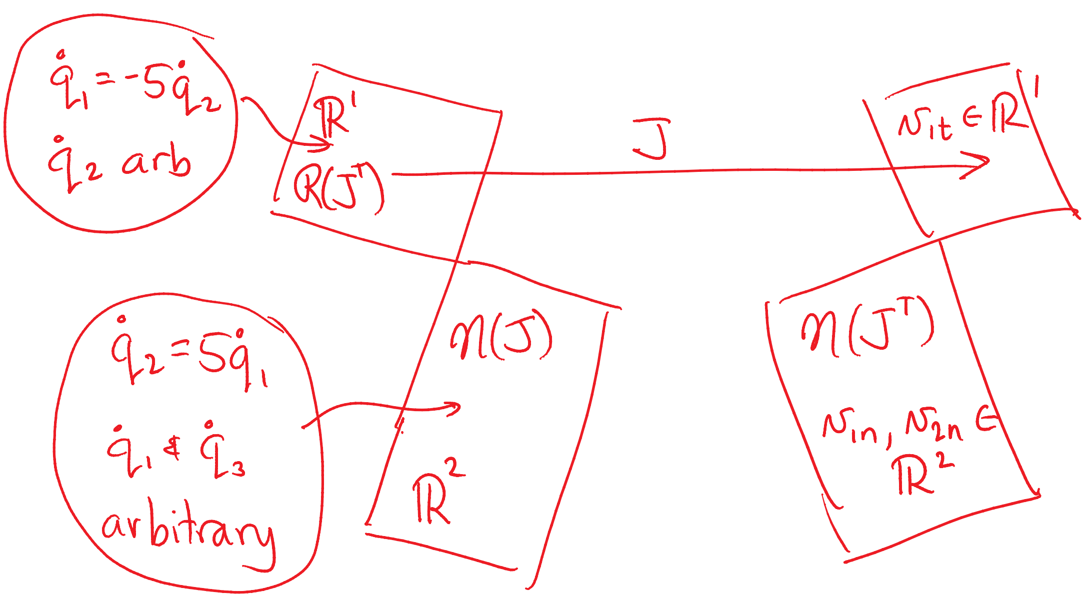
An element in $\mathcal{R}(J)$ is motion in the contact constraint directions that can be accomplished by the hand.

So, the hand can only cause contact point ① to move in the $\pm\hat{t}_1$ direction.

Force interp: The hand can control λ_{1t}, but not λ_{1n} or λ_{2n}.

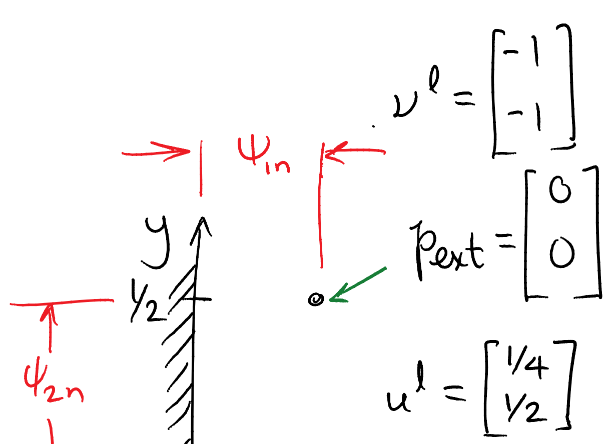
More analysis (not required for this problem).

$$\mathcal{N}(J) = \begin{bmatrix} 1 & 0 \\ 5 & 0 \\ 0 & 1 \end{bmatrix} \qquad \mathcal{N}(J^T) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$



3. A frictionless particle moves toward a corner.

Let $m = h = 1$.



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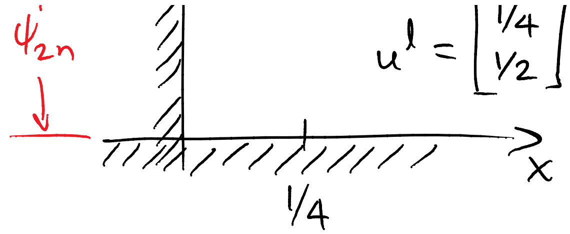


a. Set up the time-stepping LCP including both ψ_{1n} & ψ_{2n}.

$$\left.\begin{aligned}\psi_{1n} &= x \\ \psi_{2n} &= y\end{aligned}\right\} \quad \psi_n = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\Downarrow$$

$$\frac{\partial \psi_n}{\partial t} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$G_n^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

No friction $\therefore$ the eqs. become

$$\boxed{\begin{aligned}&\underline{\text{Mixed LCP}}\\ &\begin{bmatrix} 0 \\ \rho_n^{l+1} \end{bmatrix} = \begin{bmatrix} M & -G_n^T \\ G_n & 0 \end{bmatrix} \begin{bmatrix} \nu^{l+1} \\ p_n^{l+1} \end{bmatrix} + \begin{bmatrix} -M\nu^l - \cancelto{0}{p_{ext}} \\ \frac{\psi_n^l}{h} + \cancelto{0}{\frac{\partial \psi_n}{\partial t}} \end{bmatrix} \\ &0 \le \rho_n^{l+1} \perp p_n^{l+1} \ge 0\end{aligned}}$$

$$\boxed{\begin{aligned}&\underline{\text{Configuration update}}\\ &u^{l+1} = u^l + \cancelto{1}{h}\nu^{l+1}\end{aligned}}$$

Convert mixed LCP to pure LCP

$$0 = \underset{I}{M}\nu^{l+1} - \underset{I}{G_n^T} p_n^{l+1} - \underset{I}{M}\nu^l \implies \nu^{l+1} = \nu^l + p_n^{l+1}$$

$$\rho_n^{l+1} = \overset{I}{G_n^T}\nu^{l+1} + \frac{\psi_n^l}{h} \implies \rho_n^{l+1} = \nu_n^{l+1} + \cancelto{=1}{\psi_n^l / h}$$

Here is the pure LCP including both contacts.

$$\begin{bmatrix}0\\0\end{bmatrix} \le \begin{bmatrix}p_{1n}^{l+1}\\p_{2n}^{l+1}\end{bmatrix} \perp \begin{bmatrix}p_{1n}^{l+1}\\p_{2n}^{l+1}\end{bmatrix} + \begin{bmatrix}v_x^l\\v_y^l\end{bmatrix} + \begin{bmatrix}\psi_{1n}^l\\\psi_{2n}^l\end{bmatrix} \ge \begin{bmatrix}0\\0\end{bmatrix}$$

b. Temporarily ignore ψ_{2n}. Determine u^{l+1} and p_n^{l+1}.

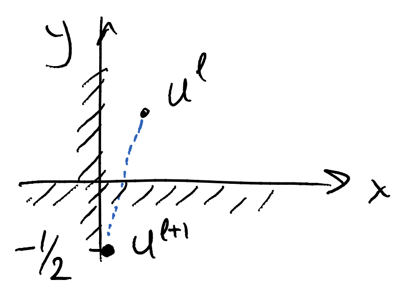
$$0 \le p_{1n}^{l+1} \perp p_{1n}^{l+1} + v_x^l + \psi_{1n}^l \ge 0$$

$$0 \le p_n^{l+1} \perp p_n^{l+1} - 1 + 1/4 \ge 0 \implies p_n^{l+1} = 3/4$$

State updates: $v^{l+1} = v^l + p_n^{l+1}$ $\quad u^{l+1} = u^l + v^{l+1}$

$$\begin{bmatrix}v_x\\v_y\end{bmatrix}^{l+1} = \begin{bmatrix}-1\\-1\end{bmatrix} + \begin{bmatrix}3/4\\0\end{bmatrix} = \begin{bmatrix}-1/4\\-1\end{bmatrix}$$

$$\begin{bmatrix}x\\y\end{bmatrix}^{l+1} = \begin{bmatrix}1/4\\1/2\end{bmatrix} + \begin{bmatrix}-1/4\\-1\end{bmatrix} = \begin{bmatrix}0\\-1/2\end{bmatrix}$$



Apparently ignoring ψ_{2n} was a bad idea.

c. For the next time step include both constraints. Compute p_n^{l+2} and u^{l+2}.

$$\begin{bmatrix}0\\0\end{bmatrix} \leq \begin{bmatrix}p_{1n}^{l+2}\\p_{2n}^{l+2}\end{bmatrix} \perp \begin{bmatrix}p_{1n}^{l+2}\\p_{2n}^{l+2}\end{bmatrix} + \begin{bmatrix}v_x^{l+1}\\v_y^{l+1}\end{bmatrix} + \begin{bmatrix}\psi_{1n}^{l+1}\\\psi_{2n}^{l+1}\end{bmatrix} \geq \begin{bmatrix}0\\0\end{bmatrix}$$

$$+ \begin{bmatrix}-1/4\\-1\end{bmatrix} + \begin{bmatrix}0\\-1/2\end{bmatrix}$$

$$\begin{bmatrix}p_{1n}^{l+2}\\p_{n}^{l+2}\end{bmatrix} = \begin{bmatrix}1/4\\3/2\end{bmatrix}$$

$$\begin{bmatrix}v_x^{l+2}\\v_y^{l+2}\end{bmatrix} = \begin{bmatrix}-1/4\\-1\end{bmatrix} + \begin{bmatrix}1/4\\3/2\end{bmatrix} = \begin{bmatrix}0\\1/2\end{bmatrix}$$ ← notice that the particle "bounced" off the horizontal obstacle, just fast enough to eliminate the penetration at the end of the time step.

$$\begin{bmatrix}x^{l+2}\\y^{l+2}\end{bmatrix} = \begin{bmatrix}0\\-1/2\end{bmatrix} + \begin{bmatrix}0\\1/2\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$$

d. Assume that Coulom friction with coefficients μ_1 & μ_2 act between the particle and the two constraint surfaces. Define E, U, G_f for this problem.

$$U = \begin{bmatrix}\mu_1 & 0\\0 & \mu_2\end{bmatrix}$$

$$\begin{bmatrix}\hat{t}_1^T\\ \hat{t}^T\end{bmatrix} \quad \begin{bmatrix}0 & 1\\0 & -1\end{bmatrix}$$

$$U = \begin{bmatrix} \cdots \\ 0 & \mu_2 \end{bmatrix}$$

$$G_f^T = \begin{bmatrix} \hat{t}_1^T \\ -\hat{t}_1^T \\ \hat{t}_2^T \\ -\hat{t}_2^T \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -1 \\ -1 & 0 \\ 1 & 0 \end{bmatrix}$$

$$E = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$$