CSCI 6490/4490: Robotics II
Mid-Term Exam
March 22, 2018

Name: ______________________________

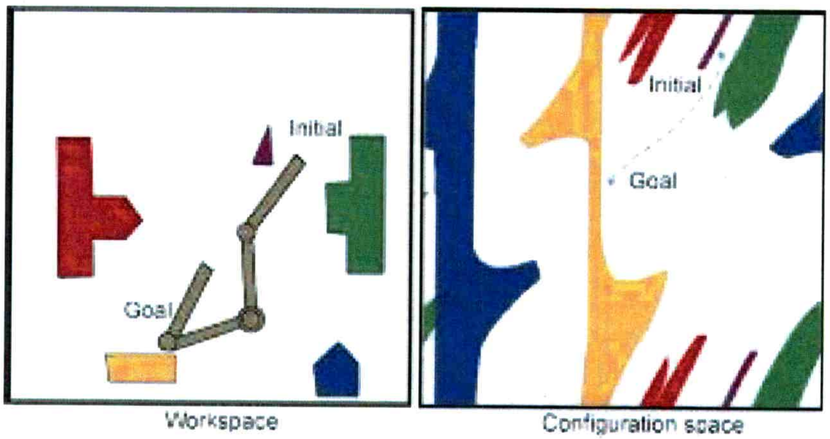


Figure 1: Work space and configuration space for a 2R manipulator.

True/False questions:

1. F (2pts) A* search with an exact cost-to-go function (perfect hueristic) is equivalent to Dijkstra's algorithm.

2. T (2pts) $C_{free} = C \backslash (C_{obs} \cup C_{contact})$, where C is the configuration space of the robot.

3. T (2pts) The interval $I = [0,\ 1]$ with endpoints identified, i.e., $[0,\ 1] \sim$, is is homeomorphic to the circle, S^1.

4. F (2pts) The Probabilistic Roadmap Method is an example of a single-shot motion planning method.

5. F (2pts) A medial axis roadmap creates the shortest possible paths for a robot moving in its workspace.

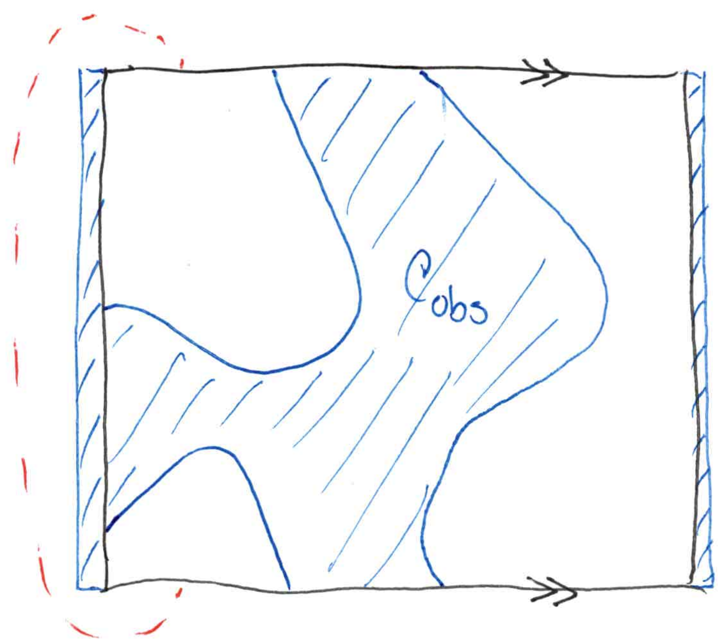
6. T (2pts) (Exact) Combinatorial motion planning algorithms can be much more efficient than sample-based methods for certain classes of problems.

7. F (2pts) Suppose orientations are represented by roll, pitch, and yaw angles. A sequence of uniform random samples of these angles is dense in $SO(3)$.

8. T (2pts) A finite number of unions and intersections of polynomial inequalities is a semi-algebraic set.

9. F (2pts) Let C-space be $I^1 \times S^1$. In the sketch below, C-space shown as a square with two edges identified (i.e., a flattened cylinder). For the C-space shown, C-free has three connected components.

It has 2 components due to the identification



Short answer questions:

11. (4pts) What two key characteristics must an algorithm have for it to be complete?

If a soln exists, the alg must find one in finite time

If a soln does not exist, the alg must report this fact in finite time.

q_I & q_G easily connected to graph.

12. (4pts) What are the main differences between algorithms that are complete and those that are probabilistically complete?

Prob. complete algs cannot determine if a soln does not exist.

If a soln exists, a prob. complete alg will find one with probability 1 if allowed to run forever.

13. (4pts) How many inverse kinematic solutions does the WAM have if a value for joint 3 is fixed? How could you characterize these solutions qualitatively?

8 IK solutions.

Elbow-up / elbow-down — 2
×
Shoulder left / Shoulder right — 2
×
Wrist-up / wrist down — 2

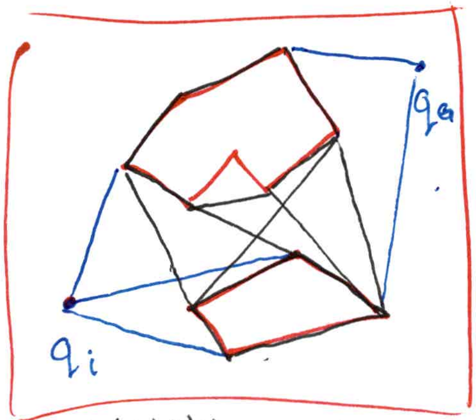
8

14. (4pts) Let R be a (3×3) rotation matrix and let h be a unit quaternion equivalent to R. Are there any other unit quaternions equivalent to the same R? If so, provide one.

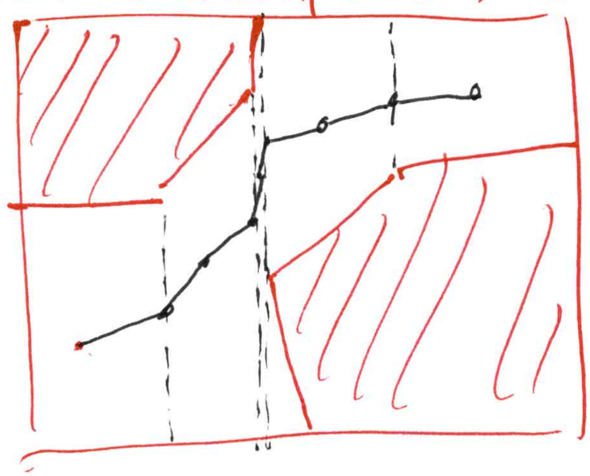
Yes. $-h$ is another $\equiv h$

15. (5pts) Describe the type of planning problem in which (exact) combinatorial motion planning methods would be preferable to sample-based methods. Illustrate your comments with a sketch.

C_{space} is planar with polygonal C_{obs}. You can use many methods: vertical decomposition, visibility graph, etc.



Visibility graph



16. (5pts) What features or characteristics of sample-based motion planning methods are responsible for probabilistic completeness?

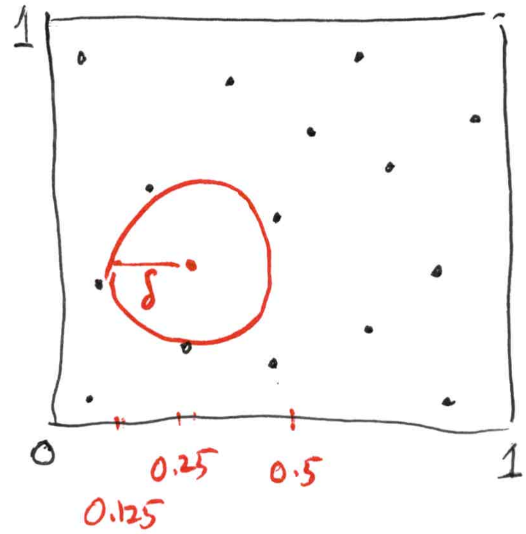
The sequence of points used must be dense on C-space (or at least on C_{free}).

The algorithm can run as long as desired, so the dispersion decreases enough to achieve the resolution necessary to find a solution if one exists.

17. (4pts) Estimate the dispersion under the L^2 norm of the samples shown in the figure below.

The largest circle's radius is δ.

Here it is about 0.13.



18. (6pts) In PRM,
a) What are the two main goals of the pre-processing phase?
b) How do these goals conflict with each other?

a.) Make a graph that reflects the connectivity (i.e. structure) of C_{free}.

The graph should be easy to connect any q_I & q_G

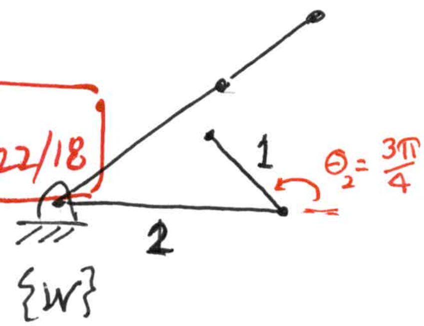
The graph should be small, so search & storage are not difficult.

b.) The alg benefits from large #s of points to achieve good coverage of C_{free}, but that increases preprocessing time & storage needs.

19. (8pts) In the current configuration, the minimum distance from the robot to any obstacle is 1. The LPM algorithm needs to test the trajectory segment shown for collision before deciding whether to insert it into the graph representing C_{free}. Determine a long interval along the trajectory segment from q_1 to q_{new} (starting at q_1) that is guaranteed to be collision free. There is not enough information to determine the longest interval, so the longest you can with the information you have.

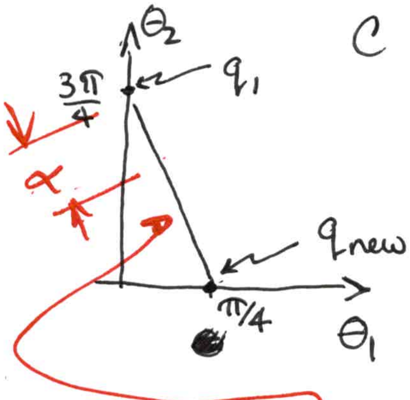
determine step

My solution follows LaValle 5.3.4 & class lecture on 2/22/18



Max distance is 1.

Conservative estimate of max displacement of any point on robot is $2\frac{\pi}{4} + 1\frac{3\pi}{4}$, if the robot follows the full traj segment



Let α be a portion of the segment. Then we need

$$\alpha \frac{5\pi}{4} \leq 1 \longrightarrow \alpha = \frac{4}{5\pi} \simeq \frac{1}{4}$$

∴ There is no need to check for collisions on the first α (fraction) of the path segment.

Defs:

guard is sample that can't see any ~~vertex~~ other guard - denote by o

connector is sample that can see at least two guards in separate components - denote by x

20. (6pts) Describe the process of constructing a visibility roadmap and sketch one for the C-space shown below.

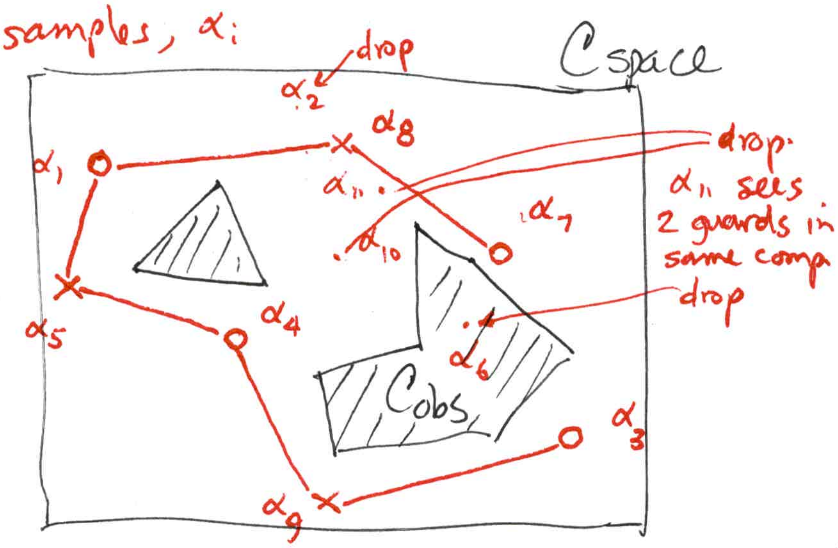
Select a dense sequence of samples, α_i

Place samples one at a time

If α_i can't see any other nodes in $\mathcal{G}$ (i.e., is a guard) keep.

If α_i is connector, add it & edges to $\mathcal{G}$.

Otherwise drop that sample.



21. (7pts) Let $\mathbb{X}$ be a topological space and let $x', x'' \in \mathbb{X}$ be points in that space. Show that $\rho(x', x'')$ defined below is or is not a metric on $\mathbb{X}$.

$$\rho(x', x'') = (\Delta x)^2 + \Delta x \tag{1}$$

where $\Delta x = x' - x''$.

Must satisfy all 4 properties, but fails several.

Symmetry:

$$(x'-x'')^2 + x'-x'' = (x''-x')^2 + x''-x'$$

$$(x')^2 - 2x'x'' + (x'')^2 + x' - x''$$

$$-\left((x'')^2 - 2x''x' + (x')^2 + x'' - x'\right)$$

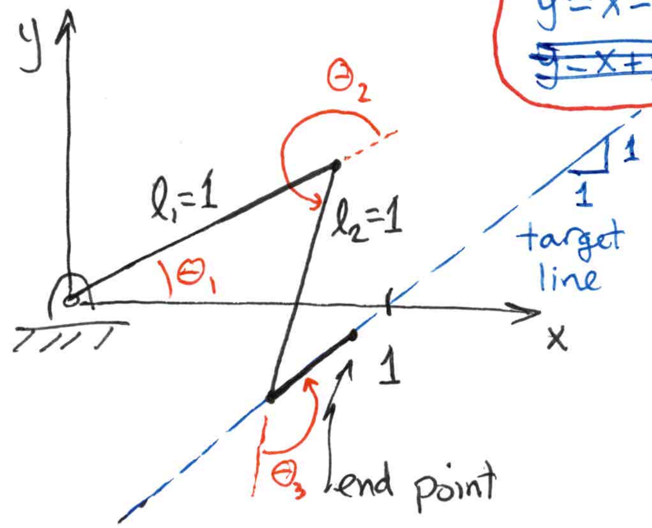
$$= 2(x'-x'') \neq 0 \qquad \therefore \text{Not a metric.}$$

22. (13pts) Solve the generalized inverse kinematics ~~problems~~ problem for the planar robot shown below. Its third link must be aligned with the target line given, but the end point may lie anywhere along that line.

$y = x - 1$ ~~$y = x + 1$~~

Constraints for this IK problem:

- jnt 2 lies on $y = x - 1$
- $\theta_1 + \theta_2 + \theta_3 \in \left\{\frac{\pi}{4}, \frac{5\pi}{4}\right\}$ ($\frac{5\pi}{4}$ ok, not req'd.)



FK: $l_1 c_1 + l_2 c_{12} = x$ (x-coord of jnt 2)

$l_1 s_1 + l_2 s_{12} = y$ (y-coord of jnt 2)

↖ substitute $y = x - 1$

Expand using identities $c_{12} = c_1 c_2 - s_1 s_2$, $s_{12} = s_1 c_2 - c_1 s_2$

Then subtract ~~first~~ 2nd eq. first eq.

$$c_1(l_1 + l_2(c_2 - s_2)) + s_1(-l_1 - l_2(s_2 + c_2)) = 1$$

← Special form with 2 solns. (elbow up & elbow down)

Choose θ_2 & solve for θ_1.

(Could also find range of θ_2 for which no soln exists)

Then $\theta_3 = \left(\frac{\pi}{4} - \theta_1 - \theta_2\right)_{\text{mod } 2\pi}$

So, there is one dof to maintain target constraint.

∴ ∞ solutions

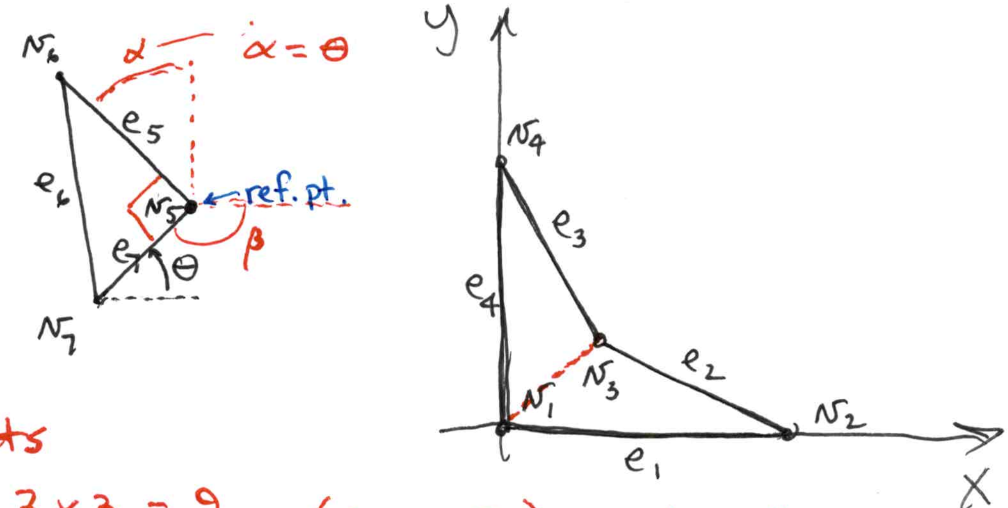
23. (12pts) For the pair of polygons in $\mathbb{R}^2$ shown in the figure below, the quadrilateral is fixed and the triangle is free to translate and rotate in the plane. Notice that v_5 is the reference point of the triangle.

a) Determine a bound on the number of facets that the C-obstacle could have in SE(2).

b) Sketch the C-obstacle under the assumption that θ is fixed at $\pi/4$ as shown in the figure.

c) Derive an edge of the C-obstacle in $SE(2)$ corresponding to vertex v_1 in contact with vertex v_5.

d) When $\theta = \pi/4$, which pair ~~or~~ of ev or ve pairs generate the facets of the C-obstacle whose intersection forms the edge from part ~~b~~ c)?



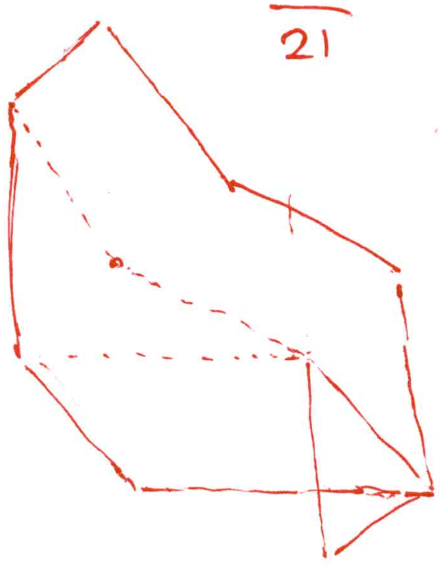
2D facets

a.) EV − 3 × 3 = 9 ($e_{5,6,7}$ v_3) are impossible

VE − 3 × 4 = 12+

21

b.)



Can do this with moving robot around or apply the star alg. on two convex parts of obstacle

c.) $x = 0$, $y = 0$, $\theta = [0, \pi]$

d.) (v_5, e_4) & (e_5, v_1)

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