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The Instantaneous Kinematics and Planning of Dextrous Manipulation

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Abstract

Dextrous manipulation is a problem of paramount importance in the study of multifingered robotic hands. In this paper, we derive in detail the kinematic relations between the finger joint velocities and object/contact velocity. The problem of dextrous manipulation is precisely formulated and cast in a form suitable for integrating relevant theory of nonholonomic motion planning, potential field methods and grasp stability to develop a general technique for dextrous manipulation planning with multifingered hands.

1 Introduction

Given an object to be manipulated by a robotic hand, the goal of dexterous manipulation planning algorithms is to generate finger joint trajectories that can drive the object to the desired configuration while simultaneously achieving the desired grasp. Various aspects of the dexterous manipulation problem have been studied by many researchers over the past decades[10], but a solution to the general problem remains elusive.

In this paper, by incorporating closed kinematic chain constraints[7] in multi-fingered hand manipulation system and the physical constraints imposed by the contact models, we derive in detail the *manipulation kinematics* which relates the object and contact movements to finger joint movements. With the results of forward and inverse manipulation kinematics, we precisely formulate the problem of dextrous manipulation planning and cast it in a form suitable for integrating the relevant theories of nonholonomic motion planning [5, 8], potential field methods [4], and grasp stability[9] to develop a general technique for dexterous manipulation with multifingered robotic hands.

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2 Mathematical Preliminaries

Following the notations in [8], we briefly review the mathematical preliminaries of rigid body motion and the kinematics of contact in this section.

We denote by $p_{ab} \in \mathrm{R}^3$ and $R_{ab} \in SO(3)$ the position and orientation of a coordinate frame B relative to another coordinate frame A, and call $g_{ab} = (R_{ab}, p_{ab}) \in SE(3)$ the Euclidean transformation of B relative to A. The velocity of B relative to A is denoted by $V_{ab} = (v_{ab}, \omega_{ab}) \in \mathrm{R}^6$. The adjoint transformation $Ad_{g_{ab}} \in \mathrm{R}^{6\times6}$ associated with g_{ab} is used to transform velocity between coordinate frames.

We parameterize the surface of a smooth object relative to a frame O by an orthogonal, right-handed coordinate chart

$$f : U \in \mathrm{R}^2 \longrightarrow S \subset \mathrm{R}^3 : \alpha = (u, v) \longmapsto f(\alpha).$$

At a point $p = f(\alpha)$, we define the Gauss frame, C, to be a coordinate frame with origin $p_{oc} = f(\alpha)$ and orientation

$$R_{oc} = \left[\begin{array}{ccc} x & y & z \end{array} \right] = \left[\begin{array}{ccc} \frac{f_u}{\|f_u\|} & \frac{f_v}{\|f_v\|} & \frac{f_u \times f_v}{\|f_u \times f_v\|} \end{array} \right] \quad (1)$$

In terms of the Gauss frame, the geometric parameters of the surface are defined by the metric tensor, $M \in \mathrm{R}^{2\times2}$, the curvature tensor, $K \in \mathrm{R}^{2\times2}$ and the torsion form, $T \in \mathrm{R}^{1\times2}$.

Consider two rigid bodies, F and O, in contact as shown in Figure 1. Let $\alpha_f = (u_f, v_f) \in \mathrm{R}^2$ and $\alpha_o = (u_o, v_o) \in \mathrm{R}^2$ be the local coordinates of F and O, respectively. Denote the geometric parameters of F by (M_f, K_f, T_f) and the the geometric parameters of O by (M_o, K_o, T_o).

A contact configuration between the two bodies is described by α_f, α_o and ψ, the angle of contact defined by the respective Gauss frames, C_f and C_o, of F and O. Let $\eta = (\alpha_f, \alpha_o, \psi) \in \mathrm{R}^5$.

Denote the contact velocity of F relative to O in terms of the local gauss frames by

$$V_c \stackrel{\text{def}}{=} V_{of}^{c_f} = [v_x,\ v_y,\ v_z,\ \omega_x,\ \omega_y,\ \omega_z]^T$$

Rolling contact constraint implies that in addition to $v_z = 0$,

$$v_x = v_y = \omega_z = 0$$

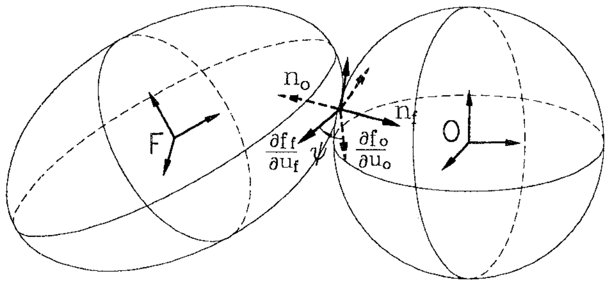

Figure 1: Motion of two objects in contact

The forward kinematic equations of contact relating the contact velocities to the rate of change of contact coordinates are given by [7]

$$V_c = \begin{bmatrix} -M_f & R_\psi M_o & 0 \\ 0 & 0 & 0 \\ R_o K_f M_f & -R_\psi R_o K_o M_o & 0 \\ -T_f M_f & -T_o M_o & 1 \end{bmatrix} \begin{bmatrix} \dot\alpha_f \\ \dot\alpha_o \\ \dot\psi \end{bmatrix} \stackrel{\text{def}}{=} J_c(\eta)\dot\eta \tag{2}$$

where $R_\psi = \begin{bmatrix} \cos\psi & -\sin\psi \\ -\sin\psi & -\cos\psi \end{bmatrix}$, $R_o = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $J_c(\eta) \in \mathbb{R}^{6\times 5}$ is referred to as the contact Jacobian.

Equation (2) can be inverted to give the rate of change of the contact coordinates, $\dot\eta$, in terms of the contact velocity[6]:

$$\begin{cases} \dot\alpha_f = M_f^{-1}(K_f + \tilde K_o)^{-1}\left(\begin{bmatrix} -\omega_y \\ \omega_x \end{bmatrix} - \tilde K_o \begin{bmatrix} v_x \\ v_y \end{bmatrix}\right) \\ \dot\alpha_o = M_o^{-1} R_\psi (K_f + \tilde K_o)^{-1}\left(\begin{bmatrix} -\omega_y \\ \omega_x \end{bmatrix} + K_f \begin{bmatrix} v_x \\ v_y \end{bmatrix}\right) \\ \dot\psi = \omega_z + T_f M_f \dot\alpha_f + T_o M_o \dot\alpha_o \end{cases} \tag{3}$$

Collectively, with the contact constraint $v_z = 0$, the above equations can be written in the form

$$\dot\eta = J_c^{-1}(\eta) V_c \tag{4}$$

where $J_c^{-1}(\eta) \in \mathbb{R}^{5\times 6}$ is the inverse of the contact Jacobian (not to be interpreted as usual matrix inverse).

3 Kinematics of Manipulation

There are 3 types of manipulation task for multifingered hand systems:

- Object Manipulation – obtain desired object velocity, and thus achieve object goal configuration
- Grasp Adjustment – obtain desired contact velocity, and thus achieve grasp goal configuration or improve the quality of grasp
- Dextrous Manipulation – achieve the above two objectives simultaneously

For a robot grasping system, only the finger joints are directly controlled by the robot system. The object and contact points are forced to move to comply with the kinematic constraints associated with the contact. Thus, the object and the contact points are 'passively' actuated, and we need to derive the **manipulation kinematics** relating object and contact movement to finger joint movement.

In the following subsections, we will develop in detail the forward and inverse kinematics for the dextrous manipulation problem. The kinematics for Object Manipulation and Grasp Adjustment can be defined and developed in the way parallel that are used for dextrous manipulation problems. For brevity, we omit the discussion here and interested readers please refer to our technical report [3]. The kinematic equations are consistent with previous research results [7, 8, 1].

3.1 Kinematics of Multifingered Robotic Hands

For a multifingered robotic hand system, set P be the palm frame, O be the object frame, and F_i, be the fingertip frame of finger i. Denote the forward kinematic map and the Jacobian of finger i by

$$g_{pf_i}(\theta_i) \in SE(3), \qquad V_{pf_i} = J_{pf_i}(\theta)\dot\theta_i$$

where $\theta_i = (\theta_{i1}, \cdots \theta_{in_i})$ is the joint angle vector of finger i.

Let $\alpha_{f_i} = (u_{f_i}, v_{f_i}) \in \mathbb{R}^2$ and $\alpha_{o_i} = (u_{o_i}, v_{o_i}) \in \mathbb{R}^2$ be the local coordinates of the contact points of finger i and object, and the corresponding Gauss frames are C_{f_i} and C_{o_i}. The contact configuration η_i is given by $\eta_i = (\alpha_{f_i}, \alpha_{o_i}, \psi_i)$, where ψ_i is the angle of contact. Contact velocity between finger i and the object is defined in terms of local gauss frames C_{f_i} by $V_{c_i} \stackrel{\text{def}}{=} V_{of}^{c_{f_i}} = (v_{ix}, v_{iy}, v_{iz}, \omega_{ix}, \omega_{iy}, \omega_{iz})$

For a m-fingered hand, let $n = \sum_{i=1}^{m} n_i$ and

$$\theta = (\theta_1, \cdots \theta_m) \in \mathbb{R}^n, \qquad \eta = (\eta_1, \cdots \eta_m) \in \mathbb{R}^{5m}$$

Given $q_i = (\theta_i, \eta_i)$, the position and orientation of the object is obtained by composing the forward kinematic map of finger i with a transformation defined by η_i,

$$g_{po} = g_{pf_i}(\theta_i) \cdot g_{f_i o}(\eta_i) \tag{5}$$

Differentiating (5) with respect to $q_i = (\theta_i, \eta_i)$ and making use of the contact Jacobian yield

$$\begin{aligned} V_{po} &= \begin{bmatrix} Ad_{g_{f_i o}^{-1}} J_{pf_i}(\theta_i) & -Ad_{g_{oc_{f_i}}} J_{c_i}(\eta_i) \end{bmatrix} \begin{bmatrix} \dot\theta_i \\ \dot\eta_i \end{bmatrix} \\ &\stackrel{\text{def}}{=} J_i(q_i)\dot q_i \end{aligned} \tag{6}$$

$J_i(q_i) \in R^{6\times(n_i+5)}$ relates the object velocity to the rate of change of the extended joint angle, $q_i = (\theta_i, \eta_i)$, and is referred to as the extended Jacobian of finger i.

By equating the right hand side of (5) and (6) for $i = 1, \cdots m$, we have the following closed-kinematic chain (or simply closure) constraints on the generalized coordinates of the system

$$\boxed{g_{po} = g_{pf_1}(\theta_1) g_{f_1o}(\eta_1) = \cdots = g_{pf_m}(\theta_m) g_{f_mo}(\eta_m)} \tag{7}$$

and

$$\boxed{V_{po} = J_1(q_1)\dot{q}_1 = \cdots = J_m(q_m)\dot{q}_m} \tag{8}$$

(7) are called the position constraints and (8) the corresponding velocity constraints.

Since the kinematic velocity constraints of contact models are more naturally expressed using the contact velocities, we will use the contact velocity instead of derivatives of contact coordinates to incorporate the physical constraints into the close kinematic chain constraints.

Physical constraints associated with contact models limit the set of admissible contact velocities, which can be expressed in the form

$$V_{c_i} = \bar{B}_i \tilde{V}_{c_i} \tag{9}$$

For example, for sliding contact, there are 5 admissible contact velocity variables:

$$\tilde{V}_{c_i} = \begin{bmatrix} v_{ix} \\ v_{iy} \\ \omega_{ix} \\ \omega_{iy} \\ \omega_{iz} \end{bmatrix} \in R^5, \qquad \bar{B}_i = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

and for pure rolling contact only 2:

$$\tilde{V}_{c_i} = \begin{bmatrix} \omega_{ix} \\ \omega_{iy} \end{bmatrix} \in R^2, \qquad \bar{B}_i = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

where $\bar{B}_i$ selects the unconstraints components of the contact velocity at contact i.

With respect to the contact velocity, the object velocity V_{po} (6) can be written as:

$$V_{po} = \begin{bmatrix} Ad_{g_{f_io}^{-1}} J_{pf_i} & -Ad_{g_{oc_{f_i}}} \bar{B}_i \end{bmatrix} \begin{bmatrix} \dot{\theta}_i \\ \tilde{V}_{c_i} \end{bmatrix} \stackrel{\text{def}}{=} \tilde{J}_i(q_i) \begin{bmatrix} \dot{\theta}_i \\ \tilde{V}_{c_i} \end{bmatrix} \tag{10}$$

where $\tilde{J}_i$ is the extended Jacobian with respect to the contact velocity.

The corresponding closed kinematic chain constraints are:

$$\boxed{V_{po} = \tilde{J}_1 \begin{bmatrix} \dot{\theta}_1 \\ \tilde{V}_{c_1} \end{bmatrix} = \tilde{J}_2 \begin{bmatrix} \dot{\theta}_2 \\ \tilde{V}_{c_2} \end{bmatrix} = \cdots = \tilde{J}_m \begin{bmatrix} \dot{\theta}_m \\ \tilde{V}_{c_m} \end{bmatrix}} \tag{11}$$

3.2 Dextrous Manipulation

By straight-forward algebraic manipulation of equation(10), we get

$$Ad_{g_{f_io}} V_{po} + Ad_{g_{f_ic_{f_i}}} \bar{B}_i \tilde{V}_{c_i} = J_{pf_i} \dot{\theta}_i \tag{12}$$

Stacking equation(12) for each finger, we can write the constraint for m-fingered hand in the matrix form that explicitly shows the dependence of the object and contact velocities on the finger joint velocities:

$$\boxed{J_{oc} \tilde{V}_{oc} = J_f \dot{\theta}} \tag{13}$$

where

$$J_{oc} = \begin{bmatrix} Ad_{g_{f_1o}} & Ad_{g_{f_1c_{f_1}}} \bar{B}_1 & & 0 \\ \vdots & & \ddots & \\ Ad_{g_{f_mo}} & 0 & & Ad_{g_{f_mc_{f_m}}} \bar{B}_m \end{bmatrix}$$

$$\tilde{V}_{oc} = \begin{bmatrix} V_{po} \\ \tilde{V}_{c_1} \\ \vdots \\ \tilde{V}_{c_m} \end{bmatrix}, \; J_f = \begin{bmatrix} J_{pf_1} & & 0 \\ & \ddots & \\ 0 & & J_{pf_m} \end{bmatrix}, \; \dot{\theta} = \begin{bmatrix} \dot{\theta}_1 \\ \vdots \\ \dot{\theta}_m \end{bmatrix}$$

The sizes of $J_{oc}, \tilde{V}_{oc}, J_f$, and $\dot{\theta}$ are, respectively, $6m \times (6 + CDOF), (6 + CDOF) \times 1, 6m \times n$, and $n \times 1$. $CDOF$ is the dimension of admissible contact velocity components,

$$CDOF = \sum_{i=1}^{m} dim(\tilde{V}_{c_i}) \tag{14}$$

For the special cases that all the contacts are pure rolling or sliding contacts, the corresponding $CDOF$ is:

$$CDOF = \begin{cases} 2m, & \text{pure rolling contacts} \\ 5m, & \text{sliding contacts} \end{cases} \tag{15}$$

The kinematic problems to be solved are

- Forward Instantaneous Kinematics

  Based on the kinematice constraints, Given joint velocity $\dot{\theta}$, are the object and contact velocity $\tilde{V}_{oc}$ uniquely determined?

  if so, the system is said to be **Kinematically-Determined.**

- Inverse Instantaneous Kinematics

  Given the desired object and contact velocity $\tilde{V}_{oc}$, is it possible to find appropriate finger joint velocity $\dot{\theta}$ to obtain such a trajectory?

  If the answer for the above question for any specified object and contact trajectory is yes , the system is said to be **Manipulable**

3.3 Forward Instantaneous Kinematics

For a given vector of joint velocities, $\dot{\theta}$, a necessary condition for the existence of $\tilde{V}_{oc}$ to satisfy equation(13) is

$$J_f\dot{\theta} \in \Re(J_{oc}), \tag{16}$$

where $\Re(J_{oc})$ is the range space of J_{oc}.

If the contacts are maintained and the contact models are correct, the above condition is automatically satisfied. A violation of the above condition indicates the joint velocities are not valid for the contact model. When $\Re(J_f) \subset \Re(J_{oc})$, any value of the joint velocity is valid.

Denote by $V^{\perp}$ the orthogonal complement of a space V. Then since $(V^{\perp})^{\perp} = V$, the equivalent condition for equation (16) is

$$J_f\dot{\theta} \in ((\Re(J_{oc}))^{\perp})^{\perp} \tag{17}$$

Recall the fundamental theorem of linear algebra[11]: $N(A^T) = (\Re(A))^{\perp}$, where $N(A)$ and $\Re(A)$ denote null space and range space of matrix A.

Suppose the singular value decomposition of matrix J_{oc} is

$$J_{oc} = U \left[\begin{array}{ccc|c} \sigma_1 & & 0 & 0 \\ & \ddots & & \vdots \\ 0 & & \sigma_r & 0 \\ 0 & \dots & 0 & 0 \end{array} \right] V^* = U \left[\begin{array}{cc} \Sigma & 0 \\ 0 & 0 \end{array} \right] V^* \tag{18}$$

where U and V are orthogonal matrices of size $6m$ and $(6+CDOF)$ respectively, $\sigma_1 \dots \sigma_r$ are the singular values of J_{oc}, and r is the rank of J_{oc}.

Suppose $U = [\ U_1 \quad U_2\ ]$, $V = [\ V_1 \quad V_2\ ]$ where $U_1 \in \Re^{6m\times r}, U_2 \in \Re^{6m\times(6m-r)}, V_1 \in \Re^{(6+CDOF)\times r}, V_2 \in \Re^{(6+CDOF)\times(6+CDOF-r)}$. Then

$$R(J_{oc}) = Span(U_1), \quad N(J_{oc}^T) = Span(U_2)$$

Thus the condition(17) can be rewritten as the following constraints:

$$\boxed{U_2^T J_f \dot{\theta} = 0} \tag{19}$$

i.e., $\dot{\theta} \in N(U_2^T J_f)$.

Suppose the set of columns of J_{fg_f} is a basis for the null space of $U_2^T J_f$, then the solution for equation(19) is

$$\dot{\theta} = J_{fg_f}\dot{\theta}_{g_f} \tag{20}$$

where $\dot{\theta}_{gf}$ is the real free parameters of finger joint velocities, and we call it generalized finger joint velocities.

Suppose the condition(16) is satisfied, then the necessary and sufficient condition to uniquely determine the object and contact velocity $\tilde{V}_{oc}$ is

$$rank(J_{oc}) = dim(\tilde{V}_{oc}) = 6 + CDOF \tag{21}$$

A necessary condition for this is $6m \geq 6 + CDOF$. Refer to equation (15), for a grasp system composed of the sliding or pure rolling contact to be kinematically determined, the minimum number of contact points, is 6 and 2 respectively.

When condition (21) is satisfied, the object and contact velocities can be determined using the generalized inverse of J_{oc} in equation(13),

$$\tilde{V}_{oc} = (J_{oc})^+ J_f\dot{\theta} = (J_{oc}^T J_{oc})^{-1} J_{oc}^T J_f \dot{\theta} \tag{22}$$

which is the least normal solution [11] for equation (13). If we didn't restrict $\dot{\theta}$ to satisfy condition (16), i.e., $J_f\dot{\theta} \notin \Re(J_{oc})$, then the least normal error of kiematic equation (13) caused by applying the solution of $\tilde{V}_{oc}(22)$, implies the kinematic constraints (13), which serve as fundamental kinematic mechanism for the robot grasping system, is violated, and thus, it is an undesirable situation that we should avoid.

If condition (16) is satisfied, we can substitute equation(20) for $\dot{\theta}$ and get the explicit dependence of the object and contact velocity on the generalized finger joint velocity $\dot{\theta}_{gf}$:

$$\tilde{V}_{oc} = (J_{oc})^+ J_f J_{fg_f}\dot{\theta}_{g_f} \stackrel{\text{def}}{=} J_{ocg_f}\dot{\theta}_{g_f} \tag{23}$$

For a kinematically-determined system, it is sufficient to use kinematic-based control to obtain a specified object/contact trajectory since actuating the finger joints to achieve the desired joint trajectories forces the object and contact velocities to be desired.

When the system is kinematically underdetermined, there are infinite solutions for equation (13). Then dynamic control need be used to remove the ambiguity of the motion of the object and contact points. Therefore, dynamics can be thought of as additional constraints which could possibly fully determine the system motions.

3.4 Inverse Instantaneous Kinematics

The objective of manipulation planning is to find the trajectory of the object, contact points, and finger joints to reach the final configuration. Since the goal is naturally specified in terms of the object and grasp configuration, it is more convenient to do the planning for the object and contact trajectory first, and then determine the desired finger joint trajectory through the kinematic constraint equation(13). Thus the inverse instantaneous kinematics problem arises: for a specified object and contact trajectory, is it possible to implement it with proper finger joint motion under the kinematic constraints? The answer must be yes for the trajectory to be feasible.

Given $\tilde{V}_{oc}$, a necessary condition for the existence of joint velocity $\dot{\theta}$ to satisfy equation(13) is

$$J_{oc}\tilde{V}_{oc} \in \Re(J_f) \tag{24}$$

If $\Re(J_{oc}) \subset \Re(J_f)$, then any value of $\tilde{V}_{oc}$ is feasible, such a system is called **Manipulable**. There are no constraints for a manipulable system on the instantaneous object and contact trajectory, since the finger joints can actuate any contact and object velocity. A sufficient condition for a system to be manipulable is that all fingers have 6 joints and all finger configurations are nonsingular, since in this case $\Re(J_f) = \Re^{6m}$.

For the general case, we can follow the steps as we have done for the finger joint velocities(16) and get similar expression as equation(20) for feasible $\tilde{V}_{oc}$:

$$\tilde{V}_{oc} = J_{ocg}\tilde{V}_{ocg} \tag{25}$$

where $\tilde{V}_{ocg}$ is the real free parameters of feasible $\tilde{V}_{oc}$ and we call it generalized object/contact velocity.

Suppose condition(24) is satisfied. If J_f has full column rank, then the joint velocity is uniquely determined using generalized inverse of J_f:

$$\dot{\theta} = J_f^+ J_{oc}\tilde{V}_{oc} = J_f^+ J_{oc} J_{ocg}\tilde{V}_{ocg} \tag{26}$$

Otherwise, there is not a unique value for $\dot{\theta}$, but rather an infinite set of possible values.

Note when the system is manipulable and kinematically determined, there will be no constraints of instantaneous manipulation planing in terms of the object and contact trajectory, and the kinematic-based control is sufficient to achieve the desired trajectory.

4 Dexterous Manipulation Planning

The objective of manipulation planning is to generate a sequence of trajectories for the fingers so that through the effects of contact constraints the system can be transferred from an initial to a goal configuration without dropping the object. The concerned state variables for dextrous manipulation are g_{po} and η. From the equations(4)(9) , we get

$$V_d \stackrel{\text{def}}{=} \begin{bmatrix} V_{po} \\ \dot{\eta}_1 \\ \vdots \\ \dot{\eta}_m \end{bmatrix} = \begin{bmatrix} I & 0 & \cdots & 0 \\ 0 & J_{c1}^{-1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & J_{cm}^{-1} \end{bmatrix} \begin{bmatrix} V_{po} \\ V_{c_1} \\ \vdots \\ V_{c_m} \end{bmatrix}$$
$$= \begin{bmatrix} I & 0 & \cdots & 0 \\ 0 & J_{c1}^{-1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & J_{cm}^{-1} \end{bmatrix} \begin{bmatrix} I & 0 & \cdots & 0 \\ 0 & \bar{B}_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \bar{B}_m \end{bmatrix} \tilde{V}_{oc}$$
$$\stackrel{\text{def}}{=} J_d\tilde{V}_{oc} \tag{27}$$

If the system is manipulable, then there will be no constraints on the object and contact velocities to be feasible. Thus we can use the above equation to do the manipulation planning directly with respect to $\tilde{V}_{oc}$. If all the contact points are sliding contact points, then $dim(V_d) = dim(\tilde{V}_{oc}) = 6+5m$, i.e. the DOF of velocity is equal to the dimension of concerned variables. While for pure rolling contact system, $dim(\tilde{V}_{oc}) = 6 + 2m < dim(V_d)$, the nonholonomic motion planning problem [8] arises.

For a general system without manipulability, we need consider the constraints on $\tilde{V}_{oc}$ to be feasible and we can further formulate the manipulation planning problem with respect to generalized object/contact velocity(25):

$$\boxed{V_d = J_d J_{ocg}\tilde{V}_{ocg}} \tag{28}$$

The desired finger joint velocity can be obtained using inverse kinematic solution (26).

Also we can formulate the problem directly with respect to the generalized finger joint velocity (23):

$$\boxed{V_d = J_d J_{ocg_f}\dot{\theta}_{g_f}} \tag{29}$$

The corresponding finger joint velocity can be obtained using equation (20).

Treating $\tilde{V}_{ocg}$ and $\dot{\theta}_{g_f}$ as the control inputs for equations(28) and (29) respectively, systems (28) (29) are referred to as standard nonholonomic systems in [5, 8]. Thus we can use general *nonholonomic motion planning* techniques to generate a trajectory for V_d, and thus, achieve the object and grasp goal configurations simultaneously.

For a manipulation task which only specifies the goal configuration for the object, we can further define the manipulation task as to (1) achieve the goal object configuration and (2) improve the quality of grasp. Thus we achieve a better coordination among the fingers. For the second objective, we need define

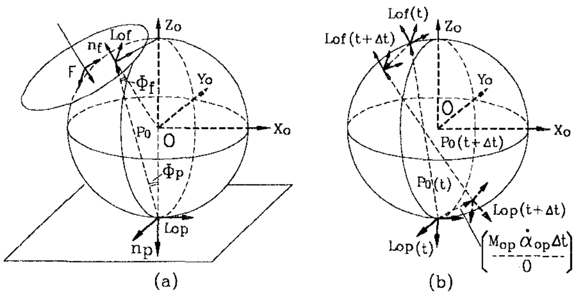

Figure 2: Grasp Angles and change of contact

a measure of quality of grasp, and one choice is grasp angles [6].

Referring to figure 2(a), let n_p and n_f be the outward surface normal of the object(ball) at the point of contact with the palm(rectangle) and the fingertip(disk), respectively.

Let $g_{l_{op}l_{of}} = g^{-1}_{ol_{op}} g_{ol_{of}} := \begin{bmatrix} R_0 & P_0 \\ 0 & 1 \end{bmatrix}$,

Then, $n_p = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$, $n_f = R_0 \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$

Define two *grasp angles* [6] by

$$\Phi_p := cos^{-1}(-n_p \bullet \frac{p_0}{||p_0||}); \quad \Phi_f := cos^{-1}(n_f \bullet \frac{p_0}{||p_0||}).$$

The grasp is force closure [9] if

$$max(\Phi_p, \Phi_f) < tan^{-1}(\mu)$$

where μ is the Comlumb friction coefficient.

We can use *potential field methods* [4] to choose the contact velocity that will minimize the grasp angles, and thus achieve optimal grasp quality.

We have applied the methodology of integrating nonholonomic motion planning techniques with potential field methods in the manipulation planning to two special but important manipulation cases: one flat finger rolling a ball on a plane and two flat fingertips manipulating a ball. The experiment results are reported in paper [2].

5 Conclusion

In this paper, we derive in detail the *manipulation kinematics* which relates object and contact movement to finger joint movement. Using results from forward and inverse manipulation kinematics, we precisely formulate the problem of dextrous manipulation planning and cast it in a form suitable for integrating relevant theories of nonholonomic motion planning, potential field methods, and grasp stability to develop a general technique for dexterous manipulation with multifingered robotic hands.

While these planing techniques work well for local motion planning, we need a complete picture of configuration space and global motion planning techniques that allow us to realize realistic, usually complicated, manipulation planning task. This forms part of our future research topics. The current theory will also be generalized to incorporate issues like hard joint limits(e.g. the workspace limits for finger joints), contact force regulation/optimization and dynamic constraints.

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