The Instantaneous Kinematics of Manipulation

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Abstract

*Dextrous manipulation planning is a problem of paramount importance in the study of multifingered robotic hands. In this paper, we show in general, that all system variables (the finger joint, object,***and** *contact velocities) need to be included in the differential kinematic equation used for manipulation planning, even if the manipulation task is only specified in terms of the goal configuration of the object or the contacts only. The dextrous manipulation kinematics that relates the finger joint movements to object* and *contact movements is derived. With the results of inverse and forward instantaneous kinematics, we precisely formulate the problem of dextrous manipulation and cast it in a form suitable for integrating the relevant theory of contact kinematics, nonholonomic motion planning, and grasp stability to develop a general technique for dextrous manipulation planning with multifingered hands.*

1 Introduction

Given an object to be manipulated by a robotic hand, the goal of dexterous manipulation planning algorithms is to generate finger joint trajectories that can drive the object to the desired configuration and/or achieve the desired grasp. There are 3 types of manipulation tasks for multifingered hand systems:

- Object Manipulation – achieve the desired object configuration without regard for contact locations;
- Grasp Adjustment – obtain desired contact locations without regard for object configuration;
- Dextrous Manipulation – achieve the goal configuration for the object and contact points simultaneously

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Velocity kinematic relationships for the first two types of manipulation tasks have been derived previously in [11, 6], respectively. The relationship for the first type of task involves only the joint and object velocities, while that of the second type contains only the joint and contact velocities. These relationships could be used as the basis of manipulatoin planning algorithms, but they lead to certain difficulties that can be avoided by using the velocity relationship implied by the third type of task, which includs joint, object*and* contact velocities. For example, developing object manipulation without considering the contact variables, the contact locations may be undesirable, since the stability of the grasp may be lost, as shown in one example in section 3. On the other hand, planning without object velocity may give a solution that causes the manipulated object collide with finger links or other objects in a crowded environment. In general, all variables in the system (the finger joint vairables, the object configuration and contact locations) need to be considered when planning any three types of manipulation planing tasks defined above. Thus the corresponding velocities should appear in the kinematic equations used for planning.

In this paper, we derive in detail the *instantaneous manipulation kinematics* relating object *and* contact movement to finger joint movement, by incorporating closed kinematic chain constraints[8] and the physical constraints imposed by the contact models(e.g. sliding, rolling etc). The kinematic equation reveals the constraints on the feasible velocities of the finger joints, the object, and the contact points to maintain a grasp with a given contact mode. We discuss the existence and uniqueness of the kinematic solution for the dextrous manipulation problem. With the results of the forward and inverse instantaneous kinematics, we precisely formulate the problem of dextrous manpulation planning and cast it in a form suitable for integrating the relevant theories of contact kinematics[7], nonholonomic motion planning [4, 9, 2] and grasp stability[10] to develop a general technique for dexterous manipulation with multifingered hands.

2 Kinematics of Manipulation

2.1 Mathematical Preliminaries

We denote by $p_{ab} \in \mathrm{R}^3$ and $R_{ab} \in SO(3)$ the position and orientation of a coordinate frame B relative to another coordinate frame A, and call $g_{ab} = (R_{ab}, p_{ab}) \in SE(3)$ the Euclidean transformation of B relative to A. The velocity of B relative to A is denoted by $V_{ab} = (v_{ab}, \omega_{ab}) \in \mathrm{R}^6$. The adjoint transformation $Ad_{g_{ab}} \in \mathrm{R}^{6\times 6}$ associated with g_{ab} is used to transform velocity between coordinate frames.

Consider two smooth rigid bodies, F and O, in contact. Let $\alpha_f = (u_f, v_f) \in \mathrm{R}^2$ and $\alpha_o = (u_o, v_o) \in \mathrm{R}^2$ be the local coordinates of the contact points on F and O, respectively, and the corresponding Gaussian frames be C_f and C_o. A contact configuration between two bodies is described by $\eta = (\alpha_o, \alpha_f, \psi) \in \mathrm{R}^5$, where ψ is the contact angle and is defined by the respective Gaussian frames C_f and C_o. $\dot{\eta}$ is called the *contact coordinate velocity.*

Denote the contact velocity of F relative to O in terms of the local Gaussian frames by

$$V_c \stackrel{\text{def}}{=} V_{of}^{c_f} = [v_x,\ v_y,\ v_z,\ \omega_x,\ \omega_y,\ \omega_z]^T$$

The contact is maintained if $v_z = 0$. The contact model will introduce more constraints, called *physical constraints*, on the contact velocity, e.g., rolling constraint requires v_x and v_y to be zero and pure rolling further requires ω_z to be zero.

The physical constraints will limit the admissible contact coordinate velocities. For example, from Montana's kinematics of pure rolling contacts[7], $(\dot{\alpha_f}, \dot{\psi})$ can be related to $\dot{\alpha_o}$ by

$$[\dot{\alpha_f}, \dot{\psi}]^T = J_{roll}\dot{\alpha_o}$$

where J_{roll} is a matrix in $R^{3\times 2}$, whose entries depend on the geometric parameters of the object and fingers. In general, the physical constraints on the contact coordinate velocities can be represented in matrix form:

$$\dot{\eta} = J_{g_c}\dot{\eta_{g_c}} \qquad (1)$$

where η_{g_c} represents the set of free parameters of contact coordinate velocity and will be referred as the generalized contact coordinate velocity; J_{g_c} is the Jacobian mapping the generalized contact coordinate velocity to contact coordinate velocity. The equation (1) for the pure rolling contacts and general contacts,i.e. contacts that admit all possible contact velocities, are

$$\begin{aligned} \text{pure rolling: } \dot{\eta} &= \left[I^{2\times 2}\ J_{roll}^T\right]^T \dot{\alpha_o} \qquad (2)\\ \text{general contacts:} \dot{\eta} &= I^{5\times 5}\dot{\eta} \end{aligned}$$

2.2 Kinematics of Multifingered Hands

For a multifingered robotic hand system, let P be the palm frame, O be the object frame, and F_i, be the frame of fingertip i. Denote the forward kinematic map and the Jacobian of finger i by

$$g_{pf_i}(\theta_i) \in SE(3), \qquad V_{pf_i} = J_{pf_i}(\theta)\dot{\theta}_i$$

where V_{pf_i} is the velocity of finger i with respect to the palm, $\theta_i = (\theta_{i1}, \cdots \theta_{in_i})$ is the joint variable vector of finger i and n_i is the number of joints of finger i. Let the contact configuration between the object and finger i be $\eta_i = (\alpha_{o_i}, \alpha_{f_i}, \psi_i)$. For an m-fingered hand, let $n = \sum_{i=1}^m n_i$ and

$$\theta = (\theta_1, \cdots \theta_m) \in \mathrm{R}^n, \qquad \eta = (\eta_1, \cdots \eta_m) \in \mathrm{R}^{5m}$$

Given $q_i = (\theta_i, \eta_i)$, the position and orientation of the object can be obtained by composing the forward kinematic map of finger i with a transformation defined by η_i,

$$g_{po} = g_{pf_i}(\theta_i) \cdot g_{f_io}(\eta_i) \qquad (3)$$

Differentiating equation (3) yields the velocity of the object with respect to the palm:

$$\begin{aligned} V_{po} &= \left[\begin{array}{cc} Ad_{g_{f_io}^{-1}} J_{pf_i}(\theta_i) & J_{c_i}(\eta_i) \end{array}\right] \left[\begin{array}{c} \dot{\theta}_i \\ \dot{\eta}_i \end{array}\right] \\ &\stackrel{\text{def}}{=} J_i(q_i)\dot{q}_i. \qquad (4) \end{aligned}$$

$J_i(q_i) \in \mathrm{R}^{6\times(n_i+5)}$ relates the object velocity to the rate of change of the extended joint coordinates $q_i = (\theta_i, \eta_i)$, and is referred to as the extended Jacobian of finger i.

By equating the right hand side of (3) and (4) for $i = 1, \cdots m$, we have the following closed-kinematic chain (or simply closure) constraints

$$\boxed{g_{po} = g_{pf_1}(\theta_1)g_{f_1o}(\eta_1) = \cdots = g_{pf_m}(\theta_m)g_{f_mo}(\eta_m)} \qquad (5)$$

and

$$\boxed{V_{po} = J_1(q_1)\dot{q}_1 = \cdots = J_m(q_m)\dot{q}_m} \qquad (6)$$

Equations (5) and (6) are called the position and velocity closure constraints,respectively.

By substituting equation(1) to the equation (4), we can incorporate the physical constraints into the kinematic chain:

$$
\begin{aligned}
V_{po} &= \begin{bmatrix} Ad_{g^{-1}_{f_i o}} J_{pf_i} & J_{c_i} J_{g_{c_i}} \end{bmatrix} \begin{bmatrix} \dot{\theta}_i \\ \dot{\eta}_{g_{c_i}} \end{bmatrix} \\
&\stackrel{\text{def}}{=} \begin{bmatrix} Ad_{g^{-1}_{f_i o}} J_{pf_i} & \tilde{J}_{c_i} \end{bmatrix} \begin{bmatrix} \dot{\theta}_i \\ \dot{\eta}_{g_{c_i}} \end{bmatrix} \\
&\stackrel{\text{def}}{=} \tilde{J}_i(q_i) \begin{bmatrix} \dot{\theta}_i \\ \dot{\eta}_{g_{c_i}} \end{bmatrix}
\end{aligned} \tag{7}
$$

where $\tilde{J}_i$ is the extended Jacobian with respect to the generalized contact coordinate velocity.

The corresponding closed kinematic chain constraints are:

$$
\boxed{V_{po} = \tilde{J}_1 \begin{bmatrix} \dot{\theta}_1 \\ \dot{\eta}_{g_{c_1}} \end{bmatrix} = \tilde{J}_2 \begin{bmatrix} \dot{\theta}_2 \\ \dot{\eta}_{g_{c_2}} \end{bmatrix} = \cdots = \tilde{J}_m \begin{bmatrix} \dot{\theta}_m \\ \dot{\eta}_{g_{c_m}} \end{bmatrix}} \tag{8}
$$

Equation(8) incoporates the closed kinematic chain constraints and the physical constraints of the contacts. It will be satisified only if the contact mode of the grasp is maintained.

2.3 Instantaneous Manipulation Kinematics

By straight-forward algebraic manipulation of equation(7), we get

$$
Ad_{g_{f_i o}} V_{po} - Ad_{g_{f_i o}} \tilde{J}_{c_i} \dot{\eta}_{g_{c_i}} = J_{pf_i} \dot{\theta}_i. \tag{9}
$$

Stacking equation(9) for each finger, we can write the constraint for an m-fingered hand in matrix form that explicitly shows the dependence of the object and contact velocities on the finger joint velocities:

$$
\boxed{J_{oc} \tilde{V}_{oc} = J_f \dot{\theta}} \tag{10}
$$

where

$$
J_{oc} = \begin{bmatrix} Ad_{g_{f_1 o}} & -Ad_{g_{f_1 o}} \tilde{J}_{c_1} & & 0 \\ \vdots & & \ddots & \\ Ad_{g_{f_m o}} & 0 & & -Ad_{g_{f_m o}} \tilde{J}_{c_m} \end{bmatrix}
$$

$$
\tilde{V}_{oc} = \begin{bmatrix} V_{po} \\ \dot{\eta}_{g_{c_1}} \\ \vdots \\ \dot{\eta}_{g_{c_m}} \end{bmatrix}, \; J_f = \begin{bmatrix} J_{pf_1} & & 0 \\ & \ddots & \\ 0 & & J_{pf_m} \end{bmatrix}, \; \dot{\theta} = \begin{bmatrix} \dot{\theta}_1 \\ \vdots \\ \dot{\theta}_m \end{bmatrix}
$$

The sizes of J_{oc}, $\tilde{V}_{oc}$, J_f, and $\dot{\theta}$ are, respectively, $6m \times (6 + CDOF)$, $(6 + CDOF) \times 1$, $6m \times n$, and $n \times 1$. $CDOF$ is the dimension of admissible contact velocity components and defined as follows:

$$
CDOF = \sum_{i=1}^{m} dim(\dot{\eta}_{g_{c_i}}). \tag{11}
$$

Pure rolling contacts have $dim(\dot{\eta}_{g_c}) = 2$; and general contacts have $dim(\dot{\eta}_{g_c}) = 5$.

From equation(10), we can extract the the dependency of the object velocity on the finger joint velocity and cast it to matrix form:

$$
G^T V_{po} = J_h \dot{\theta} \tag{12}
$$

Equation(12) is called fundamental grasp constraint in [9], in which G and J_h are called Grasp Map and Hand Jacobian,respectively. G is also known as the wrench matrix in [5].

3 Object Manipulation

The problem of object manipulation is to determine the velocity of the object relative to the palm V_{po}, without the concern for contact movement.

For an assumed set of contact locations and models, equation (12) constrains the choice of $\dot{\theta}$ for a given V_{po} or vice versa. If a solution doesn't exist, then the contact mode, positions, and/or velocity values determined by the planner must be changed. Once equation(12) is satisfied , the planner may continue to progress.

Unfortunately, the satisfication of equation(12) is not sufficient for planning. Consider the current configuration under consideration by the planner. Suppose it is stable through *force closure* but the contacts are located such that some small movement could cause a loss of force closure. In this case, equation(12) is not desirable for planning, because velocities satisfying equation(12) could destablize the grasp. For example: suppose a ball of radius 1 is grasped by two spherical fingertips of radius 0.2 and the contacts undergo pure rolling. The contact parameters, $(\alpha_f, \alpha_o, \psi)$, are $(0,0,0,0,0)$ and $(0,0,0,\frac{2\pi}{3},0)$. Suppose the coefficient of friciton is $\mu = \tan(30^o)$, then it can be shown that two contact points form a force closure grasp[10] but they are at the boundary: the grasp will not be force closure if two points move toward each other just a little bit. However, if the ball is rotated about the axis that is parallel to Z-axis and passes the point(1,0,0), it can be shown that the fingertip velocities determined by the generalized inverse of J_h are $(0,0,0,0,0,0)$ and $(-0.8660, -1.4423, 0, 0, 0, -0.2885)$. The contact velocities (w_x, w_y) are (1,0) and (1.2885,0), which will move two contact points closer to each other. Then the force closure grasp will be lost even if the equation (12) is satisified.

For the task of grasp adjustment, the kinematic equation that only includes the contact and object velocity can be derived from equation (10) and has similar problem as equation(12): it may give a solution of grasp adjustment that causes the object collide with obstacles. Therefore, in general, all system variables(the object, contact and finger joint variables) need to be considered for the manipulation planning even if the task only speicifies goal configurations for a subset of system variables, like in the cases of object manipulation and grasp adjusment. Next we will discuss in detail the dextrous manipulation kinematics which include all system states in the equation.

4 Dextrous Manipulation Kinematics

The kinematic problems to be solved are

- Forward Instantaneous Kinematics

  Based on the kinematic constraints(10), given joint velocity $\dot{\theta}$, are the object and contact velocity $\tilde{V}_{oc}$ uniquely determined?

  If so, the system is said to be **Kinematically-Determined**.

- Inverse Instantaneous Kinematics

  Given the desired object and contact velocity $\tilde{V}_{oc}$, is it possible to find appropriate finger joint velocity $\dot{\theta}$ to obtain such a trajectory?

  If the answer for the above question for any specified object and contact trajectory is yes , the system is said to be **Manipulable**

4.1 Inverse Instantaneous Kinematics

Given $\tilde{V}_{oc}$, a necessary condition for the existence of joint velocity $\dot{\theta}$ to satisfy equation(10) is

$$J_{oc}\tilde{V}_{oc} \in \Re(J_f). \tag{13}$$

where $\Re(J_f)$ is the range space of J_f.

If $\Re(J_{oc}) \subset \Re(J_f)$, then any value of $\tilde{V}_{oc}$ is feasible, such a system is called **Manipulable**. There are no constraints for a manipulable system on the instantaneous object and contact trajectory, since the finger joints can generate any contact and object velocity. A sufficient condition for a system to be manipulable is that all fingers have 6 joints and all finger configurations are nonsingular, since in this case $\Re(J_f) = \Re^{6m}$.

Denote by $V^{\perp}$ the orthogonal complement of a space V. Then since $\left(V^{\perp}\right)^{\perp} = V$, a condition equivalent to equation (13) is

$$J_{oc}\tilde{V}_{oc} \in \left((\Re(J_f))^{\perp}\right)^{\perp} \tag{14}$$

Recall the fundamental theorem of linear algebra[12]: $N(A^T) = (\Re(A))^{\perp}$, where $N(A)$ and $\Re(A)$ denote null space and range space of matrix A.

Suppose the singular value decomposition of matrix J_f is

$$J_f = U \left[\begin{array}{ccc|c} \sigma_1 & & 0 & 0 \\ & \ddots & & \vdots \\ 0 & & \sigma_r & 0 \\ 0 & \dots & 0 & 0 \end{array}\right] V^T = U \begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix} V^* \tag{15}$$

where U and V are orthogonal matrices of size $6m$ and n respectively, $\sigma_1 \dots \sigma_r$ are the singular values of J_f, and r is the rank of J_f.

Suppose $U = \begin{bmatrix} U_1 & U_2 \end{bmatrix}$, $V = \begin{bmatrix} V_1 & V_2 \end{bmatrix}$ where $U_1 \in \Re^{6m\times r}, U_2 \in \Re^{6m\times(6m-r)}, V_1 \in \Re^{n\times r}, V_2 \in \Re^{n\times(n-r)}$. Then

$$R(J_f) = Span(U_1), \quad N(J_f^T) = Span(U_2)$$

Thus condition(14) can be rewritten as the following:

$$\boxed{U_2^T J_{oc}\tilde{V}_{oc} = 0} \tag{16}$$

i.e., $\tilde{V}_{oc} \in N(U_2^T J_{oc})$.

Put a basis of the null space of $U_2^T J_{oc}$ as columns to form matrix J_{ocg}, then the solution for equation(16) is

$$\tilde{V}_{oc} = J_{ocg}\tilde{V}_{ocg} \tag{17}$$

where $\tilde{V}_{ocg}$ represents the set of free parameters of object and contact coordinate velocities, and we will refer to $\tilde{V}_{ocg}$ as the generalized object and contact coordindate velocities.

Suppose the condition(13) is satisfied, then the necessary and sufficient condition to uniquely determine the finger joint velocity $\dot{\theta}$ is

$$rank(J_f) = dim(\dot{\theta}) = n. \tag{18}$$

When condition (18) is satisfied, the finger joint velocities can be determined using the generalized inverse of J_f in equation(10),

$$\dot{\theta} = (J_f)^+ J_{oc}\tilde{V}_{oc} = (J_f^T J_f)^{-1} J_f^T J_{oc}\tilde{V}_{oc}. \tag{19}$$

If condition (13) is satisfied, we can substitute equation(17) for $\tilde{V}_{oc}$ into above equation and get the explicit dependence of the finger joint velocity on the generalized object and contact coordinate velocity $\tilde{V}_{ocg}$:

$$\dot{\theta} = (J_f)^+ J_{oc} J_{ocg}\tilde{V}_{ocg} \tag{20}$$

4.2 Forward Instantaneous Kinematics

For a given joint velocity, $\dot{\theta}$, a necessary condition for the existence of $\tilde{V}_{oc}$ satisfying equation(10) is

$$J_f \dot{\theta} \in \Re(J_{oc}) \tag{21}$$

If the contacts are maintained and the contact models are correct, the above condition is automatically satisfied. A violation of the above condition indicates the joint velocities will cause a change in the the contact mode. When $\Re(J_f) \subset \Re(J_{oc})$, any value of the joint velocity is valid.

For the general case, we can follow the steps as we have done for the generalized object and contact velocities(13) and get similar expression as equation(17) for feasible $\dot{\theta}$:

$$\dot{\theta} = J_{fg_f} \dot{\theta}_{g_f} \tag{22}$$

where $\dot{\theta}_{g_f}$ is the real free parameters of feasible $\dot{\theta}$ and will be referred as generalized finger joint velocity.

Suppose condition(21) is satisfied. If J_{oc} has full column rank, then the object and generalized contact velocity can be uniquely determined by the generalized inverse of J_{oc}:

$$\tilde{V}_{oc} = (J_{oc})^{+} J_f J_{fg_f} \dot{\theta}_{g_f} \stackrel{\text{def}}{=} J_{ocg_f} \dot{\theta}_{g_f}. \tag{23}$$

Otherwise, there is not a unique value for $\tilde{V}_{oc}$, but rather an infinite set of possible values.

For a kinematically-determined system, it is sufficient to use kinematic-based control to obtain a specified object/contact trajectory since actuating the finger joints to achieve the desired joint trajectories forces the object and contact velocities to be desired.

When the system is kinematically underdetermined, there are infinite solutions for equation (10). Then dynamic control needs be used to remove the ambiguity of the motion of the object and contact points. Therefore, dynamics can be thought of as additional constraints which could possibly fully determine the system motions.

Note that when the system is manipulable and kinematically determined, there will be no constraints of instantaneous manipulation planning in terms of the object and contact trajectory, and the kinematic-based control is sufficient to achieve the desired trajectory.

5 Dexterous Manipulation Planning

The objective of manipulation planning is to generate joint trajectories for the fingers so that the goal configuration of the object and/or contacts can be achieved, without dropping the object. The concerned state variables for dextrous manipulation are g_{po} and η. From the equations(1) , we get

$$V_d \stackrel{\text{def}}{=} \begin{bmatrix} V_{po} \\ \dot{\eta}_1 \\ \vdots \\ \dot{\eta}_m \end{bmatrix} = \begin{bmatrix} I & 0 & \cdots & 0 \\ 0 & J_{gc_1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & J_{gc_m} \end{bmatrix} \begin{bmatrix} V_{po} \\ \dot{\eta}_{g_{c_1}} \\ \vdots \\ \dot{\eta}_{g_{c_m}} \end{bmatrix} \stackrel{\text{def}}{=} J_d \tilde{V}_{oc} \tag{24}$$

If the system is manipulable, then there will be no constraints on the object and generalized contact coordinate velocities to be feasible. Thus we can use the above equation to do the manipulation planning directly with respect to $\tilde{V}_{oc}$. If all the contact points are general contacts, then $dim(V_d) = dim(\tilde{V}_{oc}) = 6 + 5m$, i.e. the DOF of velocity is equal to the dimension of concerned variables. While for pure rolling contact system, $dim(\tilde{V}_{oc}) = 6 + 2m < dim(V_d)$, the nonholonomic motion planning problem [9] arises.

For a general system without manipulability, we need to apply the constraints on $\tilde{V}_{oc}$ to equation(24) and further formulate the manipulation planning problem with respect to generalized object/contact velocity(17):

$$\boxed{V_d = J_d J_{ocg} \tilde{V}_{ocg}.} \tag{25}$$

The desired finger joint velocity can be obtained using inverse kinematic solution (20).

Also we can formulate the problem directly with respect to the generalized finger joint velocity (23):

$$\boxed{V_d = J_d J_{ocg_f} \dot{\theta}_{g_f}} \tag{26}$$

The corresponding finger joint velocity can be obtained using equation (22).

Treating $\tilde{V}_{ocg}$ and $\dot{\theta}_{g_f}$ as the control inputs for equations(25) and (26) respectively, systems (25) (26) are referred to as standard nonholonomic systems in [4, 9]. Thus we can use general *nonholonomic motion planning* techniques to generate a trajectory for V_d, and thus, achieve the object and grasp goal configurations simultaneously.

For a manipulation task which only specifies the goal configuration for the object, as we discussed in the previous section, we also need to consider the contact trajectory to maintain or optimize the grasp quality. Then we can further expand the original object manipulation task to (1) achieve the goal object configuration and (2) improve the quality of grasp. For the second objective, we need to define a measure of grasp quality and then use the gradient search to move the contact configuration to a locally optimal grasp. We have applied this methodology in the manipulation planning to two special but important manipulation cases: one flat finger rolling a ball on a plane and two flat fingertips manipulating a ball. The experimental results are reported in paper [3]. One particular point to notice is that a grasp is characterized by the contact points on the object $\{\alpha_{o_i}, i = 1...m\}$ and $\dot{\alpha_{o_i}}$ can be used as the generalized contact coordinate velocity for pure rolling contacts as indicated by equation(3). Therefore, if all contacts in a manipulation system are pure rolling and the system is manipulable, we can determine $\dot{\alpha_{o_i}}$ first by optimizing some grasp quality measure and then substitute it back to equation(24) to further determine V_d. As for the grasp adjustment, while only the goal contact points are specified, the object trajectory need to be collision free.

6 Conclusion

In this paper, we showed that in general all system variables(the velocities of the finger joints, the object and contact points) need be considered in manipulation planning even if the goal is only specified as a subset of the system states. We derived the *dextrous manipulation kinematics* which relates object *and* contact movement to finger joint movement. The existence and uniqueness of the solution for the kinematic equation of the dextrous manipulation were discussed. Using the results from forward and inverse manipulation kinematics, we precisely formulated the problem of dextrous manipulation planning and cast it in a form suitable for integrating the relevant theory of contact kinematics, nonholonomic motion planning, and grasp stability to develop a general technique for dexterous manipulation with multifingered robotic hands.

The current theory will be generalized to incorporate issues like workspace limits of hands,uncertainty and dynamic constraints. We are currently applying the analysis methods presented in paper[1] to study various properties of dextrous manipulation. While the instantaneous kinematics reveals the kinematic constraints clearly and is informative for local motion planning, we still need a representation of configuration space of the hand-object system and global motion planning techniques to enable us to implement automatic dextrous manipulation planning. This forms part of our ongoing research topics.

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