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A FRAMEWORK FOR PLANNING DEXTEROUS MANIPULATION

Jeffrey C. Trinkle
Department of Computer Science
Texas A&M University
College Station, TX 77843-3112

Jerry J. Hunter
Department of Systems and Industrial Engineering
University of Arizona
Tucson, AZ 85721

Abstract

Dexterous manipulation planning can be defined as the process of determining the joint trajectories for an articulated mechanical hand, such that, if executed, the hand would reconfigure the grasp into one known to be more desirable than the initial one. In this paper, we present a general methodology based on Desai's concept of contact formations [4] combined with a model of contact mechanics to solve the dexterous manipulation planning problem. If the model of contact mechanics supports the analysis of contact situations with multiple sliding contacts, then the method (in theory) can exploit this fact to solve problems not solvable if only rolling contacts are allowed. To highlight this feature, the method is used successfully to solve a frictionless planar example problem.

1. Introduction

Imagine a robot commanded to "clean up a spill." The robot arrives at the scene, evaluates the problem, and finds that it cannot comply with the command, because the work requires picking up slippery objects. This robot did not have an understanding of the mechanics of grasping and manipulation for the case of sliding contacts, and so failed in its mission. Had the object been sufficiently rough, the mission might have ended in success.

A more general explanation of this failure is that the robot lacks breadth of knowledge; herein lies the motivation for the research described in this paper. Our ultimate goal (which extends well beyond the scope of this paper) is to develop a complete theory of manipulation planning: one which spans the variety of environmental conditions likely to be encountered and one which can be modified to adapt to environmental uncertainties. Our approach to achieving this goal is to develop a solution method which uses the notion of contact formations [4] combined with physical models of various environments to solve manipulation planning problems. For example, if the robot is to handle compliant objects, then the physical model of rigid body interaction may be replaced by the appropriate compliant model without changing the basic structure of the planner. The only accompanying change, should be the parallel replacement of hueristics, since they are likely to differ under different environmental conditions. Toward such a universal manipulation planner, we have previously developed a quasi-static physical model to predict the motion of non-accelerating systems of rigid bodies with multiple concurrent contacts [30, 31] and algorithms for planning dexterous manipulation [27-29, 32]. Some of these algorithms depend expressly on the quasi-static model and some do not. However all of these algorithms are intended to be used to generate joint trajectories to be executed by a group of independent manipulators (*e.g.*, an articulated mechanical hand such as the Salisbury Hand).

The remainder of this Section is devoted to reviewing relevant research results. In Section 2, we describe our planning methodology in general terms and then, in Section 3, we discuss a solution to an example planning problem found by an implementation of our planning method. In Section 4, we conclude and discuss possible future enhancements of our method.

1.1. Background

The late 1960's and early 70's saw the development of the first articulated hands primarily intended for use in prosthetic applications [24, 26]. The limited success of these devices in mechanical assembly spawned the development of the well known Remote-Center-of-Compliance wrist by Whitney [33] at Draper Labs in the late 70's. Also around this time, advances in computer, sensor, and actuator technologies encouraged the birth of the second generation of mechanical hands [10, 19, 23]. Along with this second wave of hands came a much more vigorous and sustained effort to analyze these mechanisms using well known deterministic techniques of statics, kinematics, dynamics, and control. In these analyses, it was typically assumed that full knowledge of the geometry of the hand and grasped object was available and that only rolling contacts would take place, *i.e.*, it was assumed that (proper) squeezing would prevent sliding within the grasp. Under these assumptions, much progress has been made along the lines of fine manipulation planning[9, 11, 13, 14, 23] and gross manipulation planning[3, 15, 16, 20, 34].

At the same time that these detailed, deterministic analyses were being done, Mason and others were addressing the problem of acquiring a stable grasp of an object located with some degree of uncertainty on a horizontal plane. Their premise was that since sensory information is necessarily corrupt, there would always be uncertainty in the relative locations of the object and manipulator's end effector. From this fact, one must infer that contact with any (initially) ungrasped object would begin at one point, which implies that (for a sufficiently stiff system) for some period of time after initial contact, the object would slide both with respect to the table top and the end effector. In order to plan successful manipulation activities, it thus became important to understand the mechanics of sliding. These realizations motivated Mason and several other researchers [1, 2, 6, 7, 21, 25, 28] to incorporate simple models of sliding mechanics into their manipulation planning analyses. Later, the limitations of these simple models drove Peshkin to develop the Minimum Power Principle [22] to predict the quasi-static motion of a class of three-dimensional systems of bodies in sliding contact. Both Cutkosky [12] and Trinkle [30] have used Peshkin's result to formulate equations which can be used to predict or plan the motion of manipulated objects.

The work reported in this paper sides with the deterministic approach to manipulation planning. It represents a step forward in generality with respect to what has been achieved previously in the area of dexterous manipulation planning. It is unique in that it can use any model of contact mechanics without changing the basic structure of the planning method. An advantage of this feature is that one can use a model no more complicated than necessary. To illustrate our planning method, we present a simple planar example problem and the solution found under the assumption of frictionless contacts.

2. The Dexterous Manipulation Planning Problem

So that there is no confusion, we adopt the following definitions.

Hand - a set of independent manipulators controlled cooperatively to constrain and/or manipulate a common object.

Grasp - a set of contacts between the links of the independent manipulators and the common object.

Dexterous Manipulation - the act of altering the grasp of an object through the coordinated motion of the links of the hand.

Dexterous Manipulation Planning - the act of determining the joint angle and joint torque trajectories which, if performed by the hand, would effect a desired change of grasp.

In this paper we address the problem of dexterous manipulation planning. We are particularly concerned with the case for which the object must experience a "large" reorientation relative to the palm of the hand. By "large," we mean orientation changes which would require slippage at one or more contacts (*e.g.*, as studied by Brock [1] and Sastry [3]) or the use of sequence of stable finger tip grasps (*e.g.*, as studied by Grupen [8]). Reorientations considered not to be large are those which can be performed by a grasp with three rolling contacts as described in Kerr's dissertation [13].

2.1. Planning Method

One approach to solving the dexterous manipulation planning problem could be as follows. First derive the dynamic differential equations of the hand/object system, augmented with kinematic constraints and a friction model. Next define the initial and final grasps, and finally search through the space of inputs (joint torque and joint angle trajectories) to determine a solution to the two-point boundary value problem (using standard shooting and relaxation techniques).

It is our opinion that this approach is likely to fail, due to the size of the search space and the singularities (caused by gaining and losing contacts) in the dynamic system model. Therefore, we have developed an approach which effectively uses the system's singularities to naturally decompose the search space into smaller chunks which can be searched individually and independently and later combined to form a complete solution. The chunks we use are called contact formations (CF's) or, more precisely, connected contact configuration regions [4].

A CF is a qualitative description of a grasp that denotes the contacts between the fundamental elements of objects: vertices, edges, and surface patches. For the planar example shown in Figure 1, vertex 1 of the object is in contact with the third edge of the palm. Similarly noting the qualitative nature of all contacts, we may produce Table 1, the contents of which define a unique CF. Any change in these relationships defines a different CF.

The system depicted in Figure 1 has five degrees of freedom and thus its configurations may be represented as a five-dimensional configuration space [17]. The CF of the grasp shown, however, consists of four contact relationships, each of which can be

Contact Formation Information		
Object Element	Link	Hand Element
vertex 1	palm	edge 3
vertex 2	palm	edge 3
vertex 3	link 1	edge 2
edge 0	link 2	vertex 2

Table 1: Contact Formation Information.

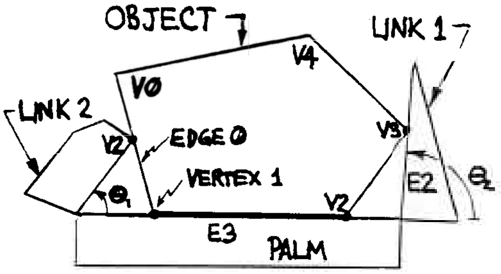

Figure 1: Example Grasp with Four Elemental Contacts.

represented as an equation in the configuration coordinates. Since the maintenance of each equation reduces the number of degrees of freedom by one, the CF may be viewed as one-dimensional curve segment in a five-dimensional space, where the curve's finite extent is due to the facts that the CF will be physically impossible to maintain for certain joint angles and that the joints are assumed to have motion limits.

The basic idea of our approach is to consider the initial and final grasps as CF's rather than as points in configuration space. Since Desai showed that configuration space is partitioned by CF's, we may search the set of all possible CF's (a finite set) for a feasible sequence which transforms the initial CF into the goal CF. Having found such a sequence, the chosen physical model of object interaction may be used to check its feasibility. This physical model is also used to determine the system inputs (and the corresponding path through configuration space) as precise continuous functions suitable for the control of an actual dexterous hand. Thus planning takes place at two levels of detail: the coarse level in which CF's are of prime importance and a fine level in which continuous trajectories are computed to carry the system through a chosen sequence of CF's. One last general note about our planning method is that the coarse-level planning need not be complete before beginning fine-level planning. In fact, it is often useful to intermingle the two planning levels so that infeasible sequences of CF's are not considered.

2.2. Plan Generation

Our method of generating dexterous manipulation plans is best described through explanation of Figure 2 below. There are two CF-trees: the *initial tree* and the *goal tree*. Each node represents a CF and each arc represents an input trajectory which transforms the grasp from a specific configuration in one of the CF's into a specific configuration in the other CF. To find a dexterous manipulation plan, the trees are grown (perhaps constrained by the physical model of the hand/object system) until they contain a common CF (CF_7 and CF_{-4} in Figure 2). At this stage, the trees contain trajectory information to transform the initial grasp into the grasp corresponding to the point I in the common CF and from the point G to the goal grasp. Only the trajectory connecting I to G is missing. If such a trajectory can be found, then the dexterous manipulation planning has a solution and the trajectories contained in the arcs can be combined serially to produce the complete trajectory required to carry out the desired dexterous operation. If such a trajectory cannot be found and the trees are not complete, then tree building must continue. Otherwise, no solution exists. An algorithm describing this solution technique follows.

1. *Initialize the trees as single nodes, then go to step 6.*
2. *If the trees are complete, then go to step 9.*
3. *Select current node.*
4. *Find all children.*
5. *Add new nodes and arcs.*
6. *If a common CF does not exist, then go to step 2.*
7. *If a trajectory from I to G cannot be found, then go to step 2.*
8. *Join the arcs' input trajectories to generate the hand's complete input trajectories.*
9. *Quit*

3. Results

Our planning technique was applied successfully to the planar dexterous manipulation planning problem illustrated below. Figures 3a and 3b show the initial and goal grasps. The objective was to find joint trajectories to transform the initial grasp into the final grasp while enforcing the following assumptions and restrictions: the bodies were rigid and moved slowly enough to neglect dynamic effects, the contacts were frictionless, the palm was fixed in the plane, and gravity acted down the page.

The system considered has five degrees of freedom which suggests a five-dimensional configuration

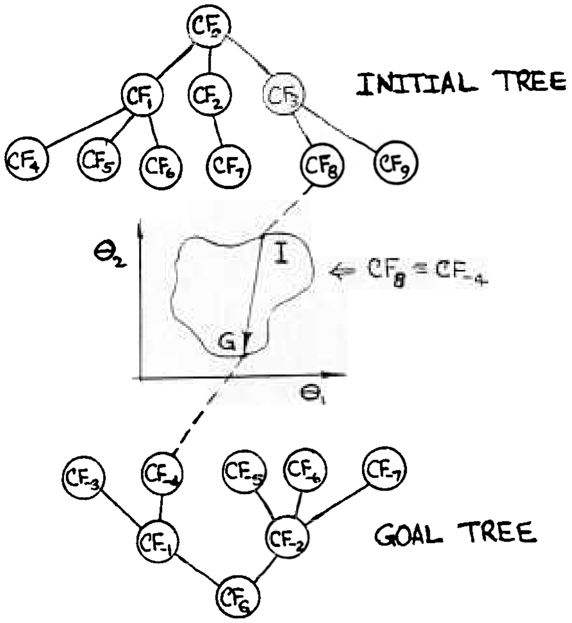

Figure 2: Planning with CF-Trees.

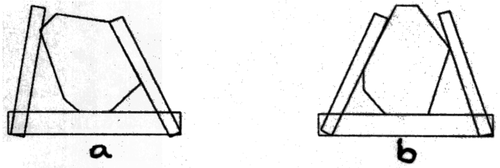

Figure 3: Initial and Goal Grasps.

space for planning. However, we can reduce the dimension of the configuration space to two, because our assumptions provide a means to compute the locally unique solution of the object's configuration given the joint angles. In other words, there exists a mapping between joint space and object configuration space which is locally one-to-one and onto (except for rarely occurring pathological cases). The one-to-one and onto properties of the mapping can be seen by noting that for a stable grasp, the object will respond to changes in the joint angles by moving to minimize its potential energy. Only for large joint motions (and pathological cases) will there be multiple solutions of the objects configuration. Thus all planning for this problem can be done in joint space. We emphasize that the reduction in the dimension of the search (input) space is due to the model of physics chosen, not to our planning technique. Subsequent partitioning of the search space is a feature of our technique.

Figure 4 shows a sequence of simulated frames of action which transform the initial grasp into the desired final grasp. These frames represent the dexterous manipulation plan found in approximately one minute of cpu time on an IBM compatible computer (80386/80387 operating at 16.67 MHz). Figure 5 shows the corresponding connected initial and goal trees.

In Figure 4, Frame 1 corresponds to the root node of the initial tree (see Figure 5).

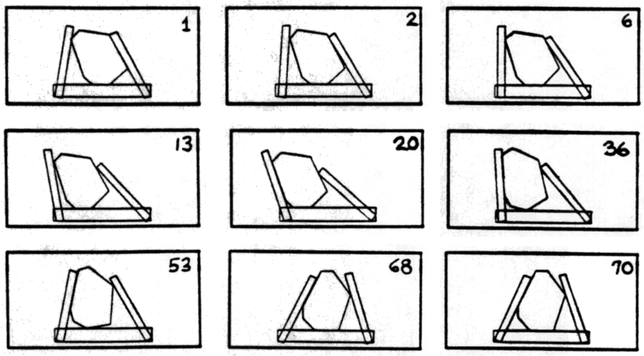

Figure 4: Frames of Action of Planned Manipulation.

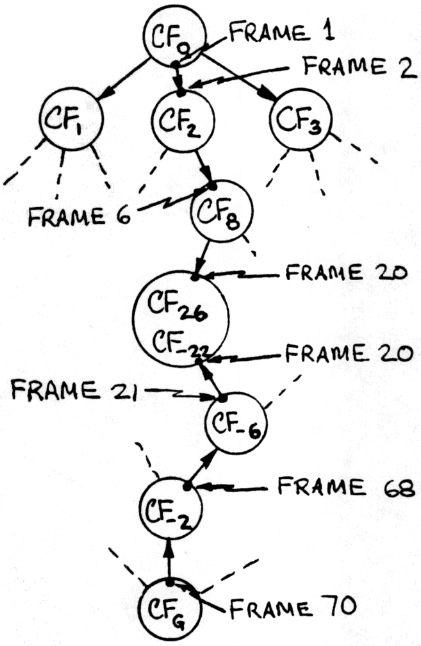

Figure 5: CF-Tree Corresponding to Planned Manipulation.

Note that since there are five elemental contact, any feasible motion of the system must break at least one contact. Thus Frame 2 shows a grasp which belongs to a different CF, namely CF_2, and the infinitesimal joint trajectories required to manipulate the grasp from Frame 1 to Frame 2 are stored in the arc connecting CF_0 and CF_2 in the initial tree. Since CF_2 consists of four contacts, it can be viewed as a curve in joint space whose end points correspond to Frames 1 and 20 and nodes CF_2 and CF_{26}. Notice however, that between CF_2 and CF_{26} is node CF_8 which does not correspond to a new CF according to our definition. In examining the grasps (not all shown in Figure 4), one can determine that the grasps in Frames

2-5 have form closure and those in Frames 6-19 have force closure [18]. This qualitative difference in grasp stability is important to our procedure for generating nodes, therefore each CF was partitioned on this basis. With this partitioning in mind, we see that the arcs from CF_2 to CF_8 and from CF_8 to CF_{26} contain trajectory information for manipulation from Frame 2 to Frame 6 and from Frame 6 to Frame 20.

The trees join at CF_{26} and CF_{-22}. These CF's are identical, single-point CF's, because they both represent the five-contact grasp shown in Frame 20. As a result, this point is both I and G as defined above. As such, the degenerate path connecting the points is known. The arcs and nodes of the goal tree are interpreted just as those in the initial tree. The arc from CF_{-22} to CF_{-6} represents the infinitesimal trajectories required to break the left-most contact on the palm in Frame 20. The arc from CF_{-6} to CF_{-2} represents the joint trajectories to tip the object over until a new palm contact in gained in Frame 68. And the last arc represents the joint trajectories required to slide the object along the palm to achieve the goal grasp.

Our assumption that contacts were frictionless made all arcs in the trees, or equivalently all trajectory segments, reversible. This allowed both trees to be grown with exactly the same procedures and also allowed the trajectories to be reversed to manipulate the object back into the initial grasp from the goal grasp. However, if friction were present, or momentary instabilities were allowed, then the arcs would not be reversible, so goal tree would need to be grown by backward chaining [5].

One question which might be asked in light of reversibility is why does the object slide toward the left (Frames 1-20) while the fingers push left and then when the fingers push toward the right (Frames 20-68), the object tips instead of reversing its course? The reason is that in Frame 20 there are five contacts. For manipulation to precede from there, one contact must break. Breaking a contact while maintaining the other four requires a specific ratio of joint velocities and joint torques. Thus achieving the grasp in Frame 19 from the one in Frame 20 requires one velocity ratio, while initiating tipping to move from Frame 20 to Frame 21 requires another. This fact appears as a corner in the joint space trajectory (see Figure 6) at the point labeled "time = 20". Note that similar corners corresponding to frames 44 and 68 are also evident; the corner at time 68 is much like that at time 20, but the corner at time 44 is caused by a discontinuity in the contact normal as the right finger tip crosses a vertex of the object.

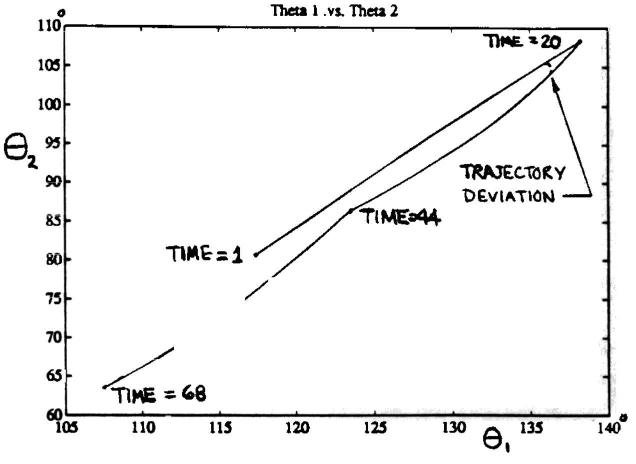

Figure 6: Joint-Space Trajectory Corresponding to Planned Manipulation.

Another valid question is how can one hope to control the finger joints to follow the required joint trajectories accurately enough to perform the planned manipulation? The answer to this question lies mainly in Figure 7,

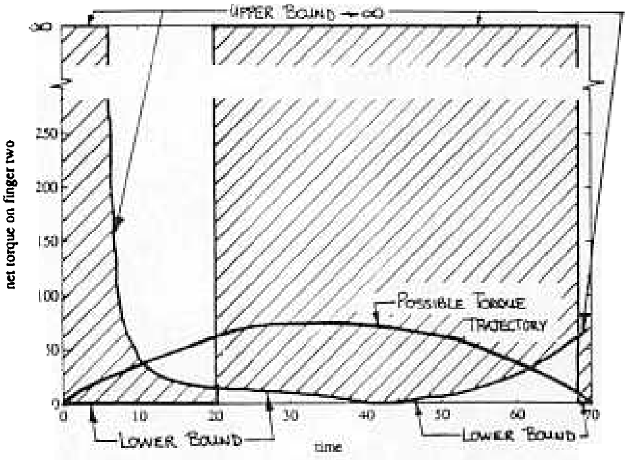

Figure 7: Joint Torque Limits for the Left Finger

which shows the upper and lower bounds on the magnitude of the joint torques for the left finger. To exactly execute the planned manipulation, the joint torques of the left finger must remain within the bounds shown, which would be impossible at times 20 and 68. However, this problem can be side-stepped by realizing that if the torque in Figure 7 were to exceed its upper bound between times 6 and 19, the left-most contact on the palm would break, effectively allowing the specific grasp shown in Frame 20 to be skipped. Thus instead of attempting to control the torque trajectory to pass through the specific torque value at time 20, one could purposely squeeze more tightly as time 20 was approached as indicated by the "Possible Torque Trajectory" in Figure 6. The effect on the joint space trajectory is shown as a dashed curve near the corner at time 20 in Figure 7. To avoid a similar problem at time 68, the hand could be controlled to squeeze less tightly, so that the object's weight would spread the fingers and lead to a second contact on the palm. In other words, before achieving the second palm contact at time 68, the grasp could be relaxed, allowing the object to fall into the goal grasp configuration. These considerations lead us to believe that the example manipulation plan could be executed by a hand with a stiff position control loop closed around one finger and a torque control loop closed around the other.

One should note that the changes to the joint position and torque trajectories suggested above should exist (in theory) in the trees as alternative paths, however, our current implementation only builds the trees until the first solution is found. No attempt to find the "best" solution is made. One way to find the alternative solution discussed would be to dissallow nodes corresponding to five-point contacts.

4. Conclusions and Future Work

We have presented the conceptual framework for a general dexterous manipulation planner in a deterministic setting. Based on our methodology, a planner would effectively solve two-point boundary value problems by using contact formation transitions to discretize the configuration space of the hand/object system (much like relaxation techniques typically used to solve standard two-point boundary value problems). Within each discrete cell, or contact formation, a model of contact mechanics is used to generate trajectories joining the cells and building a contact formation tree. If a solution exists, the tree grows until it contains a path from the initial grasp to the goal grasp. Then the individual input trajectories (assigned to the arcs of the tree) are combined to generate the complete manipulation trajectories. Precisely how best to grow the tree is an open question.

The framework is general enough to be used with complex mechanical hands and combined with any model of contact mechanics; dynamic or quasi-static, compliant or rigid. However, implementation of this planning methodology in an n-dimensional space where "n" could be as large as 20 will be nontrivial. Also, while we have discussed our method only with reference to the dexterous manipulation planning problem, the framework can be applied to mechanical assembly planning problems, too.

We justify the determinism of our planning methodology through our contention that the abilities of deterministic planning techniques serve as upper bounds on what can be achieved when uncertainty is considered. However, since one must eventually come to grips with uncertainty (in a formal way), work is continuing on contact formation representations that can be used when physical parameters are uncertain and on the parametric sensitivity of a quasi-static model of contact mechanics.

Other open issues include the questions of existence and optimality. For example, it is important to know that a particular dexterous manipulation problem has no solution under a specific set of assumptions and that under another set it does. Also, if at least one solution exists, we would like to know if it is optimal. This requires first, knowledge of all trajectories which transform the initial grasp to the goal grasp and second, a definition of optimality, which could conceivably be goal dependent.

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