A Quasi-Static Analysis of Dexterous Manipulation with Sliding and Rolling Contacts

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Abstract

Dexterous manipulation refers to the skillful execution of object reorienting and repositioning maneuvers, especially when performed by an articulated mechanical hand. Kinematic analyses of dexterous manipulation are limited by their omission of contact forces. In this paper, Peshkin's minimum power principle [15] for quasi-static systems is used to combine force and kinematic relationships into a nonlinear mathematical program called the *forward object motion problem*. Given the joint velocities of the robot's hand and arm, the solution of the forward object motion problem predicts not only the velocity of the object (as done by kinematic analyses), but also the contact forces. In kinematic analyses one must guess as to the nature of all contact interactions (*i.e.* sliding, rolling or separating). The solution of the forward object motion problem definitively determines these interactions and the contact forces as a byproduct of determining the velocity of the manipulated object.

1. Introduction

The first example of intelligent robotic manipulation was demonstrated by Paul in 1970 [14] using a parallel-jawed gripper mounted on the Stanford Arm. The robot reoriented and stacked a number of colored blocks through a sequence of grasps, reorienting arm motions and releases. Paul's work highlighted the fact that robot arms have relatively coarse resolution and require relatively large joint motions to effect relatively small object motions. One solution to these problems is to mount an articulated mechanical hand on the end of the arm. A robot so equipped can achieve "fine" motion control while reducing concern for arm collisions by initially positioning the arm and then immobilizing it while the fingers manipulate. Recognition of this fact led firstly to the design and manufacture of several articulated mechanical hands and secondly to the study of the kinematics of fine manipulation. The shortcomings of these analyses (discussed in the next two subsections) motivated the work presented here on the *forward object motion problem*.

For the purposes of this paper, the forward kinematic problem for dexterous hands refers to solving kinematic relationships in such a way as to yield a system of equations which describes the velocity and angular velocity of the manipulated object as functions of the robot's joint velocities. The inverse kinematic problem results in a system of equations describing the joint velocities as functions of the object's translational and angular velocities. In both cases it is assumed that the positions of the contact points are known. Also, by robot is meant an articulated mechanical hand affixed to the distal end of a robotic arm.

1.1. Rolling Manipulation

The inverse kinematic problem was first applied to dexterous manipulation by Okada [13] to plan joint trajectories to control an eleven-jointed, three-fingered hand to thread a nut onto a bolt. In his solution he imposed the constraint that the finger tips roll (and never slide) on the nut. Because Okada's results only applied to spherical and planar surfaces, Kerr [9] and Ji [8] derived systems of differential equations relating the object velocity to the joint velocities for arbitrarily shaped finger tips rolling on an arbitrarily shaped object. The equations are quite complex. For example, application of Kerr's results to manipulation with three fingertips rolling on an object yields 27 simultaneous, nonlinear, time-varying, coupled, first order differential equations in the following form

$$\mathbf{P}\,\dot{\psi} = \mathbf{Q}\,\mathbf{v}_{ob} \tag{3}$$

where $\mathbf{P}$ is a 27 x 17 matrix, $\mathbf{Q}$ is a 27 x 6 matrix, $\dot{\psi}$ is the time derivative of the vector of joint angles and surface parameters of the object and fingers and $\mathbf{v}_{ob}$ is the velocity of the object. Since there are 27 equations and 23 unknowns, ten dependent equations must be identified (if possible) and removed before one may specify either the desired trajectory of the object or the desired trajectories of six joints (the other joint trajectories are determined by the closed-loop kinematic structure). If elimination of the proper number of equations were possible, the resulting equations, while computationally expensive, offer the freedom to solve either the forward or the inverse kinematic problem where in both cases the initial grasp configuration is assumed to be known. However, as pointed out by Ji [8], joint angle limitations, geometric interferences or insufficient actuator torques may cause one or more contacts to slip. In this case the kinematic equations must be valid for rolling and slipping and additionally a means is required to determine the nature of the contact interactions (*i.e.*, rolling, sliding or separating).

1.2. Sliding Manipulation

Attempts to quantify the effects of sliding during manipulation were first made by Mason [11] for the case of a pushed objects sliding quasi-statically in a horizontal plane. Peshkin [16] found quantitative bounds on several of Mason's qualitative results by considering all possible supporting contact distributions. Also working in the plane, Brost and Erdmann developed techniques to remove all uncertainty in the orientation of planar objects; the former through squeezing operations with a parallel-jawed gripper [3] and the latter by planning a sequence of tilting operations of tray containing the object [4]. Another planar manipulation problem was studied by Fearing who developed of an algorithm to enable the Salisbury hand to "twirl a baton" in a vertical plane [5, 6]. Using the Salisbury Hand, Brock demonstrated manipulation with "controlled slip." [2] For manipulation of a three-finger tip grasp, his method can be viewed as choosing two finger tip contacts to define an axis of rotation. Those two fingers apply a somewhat larger normal force than the third finger tip which is dragged across the object so that its friction force causes a moment and induces rotation about the axis defined by the two finger tips. Brock's main contribution was to determine the appropriate internal grasp forces to ensure success of the desired rotation. His method's main drawbacks are complexity for more than three contacts and difficulty of inversion. One other study on contact slippage was undertaken be Nguyen [12]. He was concerned with the stability of static grasps for which manipulation occurred *passively*. That is, the object's motion resulted from the deformation of the hand in response to changes in the external wrench applied to the object. Active manipulation was not considered.

The only work on sliding manipulation for general three-dimensional objects has been done by Ji [8] and Trinkle [18]. Ji's dissertation contains results for fingertip grasps analogous to those developed by Kerr [9] for fingertips with rolling contacts. His result's major weakness is that it relies on the contacts constraining the object such that there is a unique kinematically admissible motion. Trinkle [17] developed the frictionless object motion problem specifically to predict the motion of the grasped object in response to the motion of the robot when there is more than one (possibly an infinite set of) kinematically admissible motion.

1.3. Short-Comings and Proposed Remedies

Kinematic analyses are not entirely suitable for application to many manipulation problems, because they do not include knowledge of the contact forces. Thus one must guess as to the contact interactions and/or augment the kinematic analysis with force information. Complete dynamic analyses are not always the correct choice either due to their complexity. A specific application which highlights this point is mechanical assembly. Assembly operations for complex parts are usually performed carefully and slowly. Under these conditions, the assembly process may be approximated as quasi-static with friction forces and compliance effects dominating the motion of the mating workpieces.

Current knowledge in dexterous manipulation is lacking primarily in two regards:

1. Almost all results for three-dimensional manipulation apply to grasps with three fingertips. Contacts between the object and proximal links of the hand and other objects in the environment have been neglected.
2. Available results are best suited to applications where the effective coefficients of friction acting at the contacts are very high (rolling results of Ji and Kerr) or very low (sliding results of Trinkle). No technique exists with which one may study quasi-static, dexterous manipulation when the coefficients of friction are moderate, so that rolling and sliding occur simultaneously.

Some drawbacks of attempting to utilize currently available research results can be seen by considering the following scenario. Suppose that to test proposed mechanical assembly plans before performing them on the shop floor, the plans were simulated using either Trinkle's frictionless result or Ji's sliding kinematic result. Such a simulator, if based on either result, could erroneously predict successful assembly motions when in fact friction would cause the workpieces would jam. Additionally, if the simulator were based on the former result, contact forces and required actuator torques would consistently be underestimated, leading to excessive wear and part breakage. An assembly simulator could not be based on any of the other results discussed above, because they all assume rolling contacts: mechanical assembly invariably involves sliding contacts.

In the following section, the forward object motion problem is introduced. For the case of quasi-static manipulation (*e.g.*, mechanical assembly), it provides remedies for the two major short-comings noted above: allowing the object to contact any link of the robot and any other body in the environment; and providing an analytical tool valid for any value of friction coefficients. The forerunner of the forward object motion problem is the frictionless object motion problem which was developed in [17].

2. The Forward Object Motion Problem

The forward object motion problem uniquely combines the kinematic constraints and the equations governing the forces of quasi-static manipulation into an optimization problem. This optimization problem represents the instantaneous equations of motion of the manipulated object. The object may contact any link of the robot and any body in the environment; moving or stationary. The input to the forward object motion problem is the vector of joint velocities, the current contact configuration and the effective coefficients of friction acting at each contact. The solution yields the contact forces, the velocity of the object and the nature of the contact interaction at each contact, *i.e.*, sliding, rolling or separating. Because both kinematic and force information is included, contact forces and jamming conditions may be predicted; a task which purely kinematic analyses cannot perform. If all friction coefficients are zero, then the optimization problem reduces to a linear program [17].

In the following derivation of the forward object motion problem, two fundamental assumptions are made: first, all bodies are rigid and second, the manipulation system obeys Peshkin's minimum power principle [15]. Roughly speaking the "...minimum power principle states that a system chooses at every instant the lowest energy or 'easiest' motion in conformity with the constraints." This principle applies only to quasi-static systems subject to forces of constraint (*i.e.*, normal forces arising due to contacts among rigid bodies), Coulomb friction forces and forces independent of velocity. For this principle the power is defined as

$$P_{zc} = -\sum_i \mathbf{f}_{zci} \cdot \mathbf{v}_i \tag{3}$$

where $\mathbf{v}_i$ is the velocity of the i^{th} point of application of external forces and $\mathbf{f}_{zci}$ is the sum of the external forces, excluding constraint forces, applied to the i^{th} point. Included in P_{zc} are the friction and gravitational forces. The normal forces at the contacts are omitted. Thus P_{zc} is only a fraction of the total power.

The wrench $\mathbf{w}_i$, applied to the object through the i^{th} point contact with friction can be written as the product of the i^{th} contact's unit wrench matrix $\mathbf{W}_i$ and the wrench intensity vector $\mathbf{c}_i$ as

$$\mathbf{w}_i = \mathbf{W}_i\ \mathbf{c}_i \qquad i = 1, \ldots, n_c \tag{4}$$

where n_c is the number of contact points,

$$\mathbf{W}_i = \begin{bmatrix} \hat{n}_i & \hat{o}_i & \hat{a}_i \\ \mathbf{r}_i \times \hat{n}_i & \mathbf{r}_i \times \hat{o}_i & \mathbf{r}_i \times \hat{a}_i \end{bmatrix} \qquad \mathbf{c}_i = \begin{bmatrix} c_{in} \\ c_{io} \\ c_{ia} \end{bmatrix}$$

$\mathbf{r}_i$ is the position of the i^{th} contact point, $\hat{a}_i$ is the contact's unit normal directed inward with respect to the object, $\hat{n}_i$ and $\hat{o}_i$ are orthogonal unit vectors defining the contact tangent plane and the elements of $\mathbf{c}_i$ are the magnitudes of the i^{th} contact force in the $\hat{n}_i$, $\hat{o}_i$ and $\hat{a}_i$ directions. Including all of the contacts, the equilibrium relationships can be written and partitioned as follows

$$\begin{bmatrix} \mathbf{W}_n & \mathbf{W}_o & \mathbf{W}_a \end{bmatrix} \begin{bmatrix} \mathbf{c}_n \\ \mathbf{c}_o \\ \mathbf{c}_a \end{bmatrix} = -\mathbf{g}_{ext} \tag{5}$$

where $\mathbf{g}_{ext}$ is the gravitational wrench acting on the object,

$$\mathbf{W}_p = \begin{bmatrix} \hat{p}_1 & \hat{p}_2 & & \hat{p}_{n_c} \\ \mathbf{r}_1 \times \hat{p}_1 & \mathbf{r}_2 \times \hat{p}_2 & & \mathbf{r}_{n_c} \times \hat{p}_{n_c} \end{bmatrix}$$

$$\mathbf{c}_p = \begin{bmatrix} c_{1p} \\ c_{2p} \\ \cdot \\ \\ c_{n_c p} \end{bmatrix}$$

where $p \in \{n, o, a\}$. This partitioning of the wrench matrix allows us to separate the sum of the friction wrenches $\mathbf{W}_n\ \mathbf{c}_n + \mathbf{W}_o\ \mathbf{c}_o$ from the sum of the contact normal wrenches $\mathbf{W}_a\ \mathbf{c}_a$. The equation for P_{zc} may now be written as follows

$$P_{zc} = -\dot{\mathbf{q}}_{ob}^{\mathrm{T}} \{\mathbf{g}_{ext} + \begin{bmatrix} \mathbf{W}_n & \mathbf{W}_o \end{bmatrix} \begin{bmatrix} \mathbf{c}_n \\ \mathbf{c}_o \end{bmatrix}\} \tag{6}$$

where $\dot{\mathbf{q}}_{ob}$ represents the object's linear and angular velocities and the superscript $^{\mathrm{T}}$ denotes matrix transposition. Given the joint velocities of the robot (hand and arm) $\dot{\theta}$ the velocity of the object may be found by minimizing P_{zc} subject to rigid body velocity constraints (*i.e.*, kinematic constraints) and Coulomb friction constraints. The velocity constraints disallow interference between bodies and may be written as

$$\mathbf{W}_a^{\mathrm{T}}\ \dot{\mathbf{q}}_{ob} \geqslant \mathbf{L}_a\ \dot{\theta} \tag{7}$$

where $\mathbf{W}_a^{\mathrm{T}}\ \dot{\mathbf{q}}_{ob}$ and $\mathbf{L}_a\ \dot{\theta}$ are the vectors of the normal velocity components of the contact points on the object and the hand respectively and $\mathbf{L}_a$ is a partition of the transmitted Jacobian [17]. The Coulomb friction constraints require that the i^{th} contact force lie within the friction cone given by

$$c_{in}^2 + c_{io}^2 \leqslant \mu_i^2\ c_{ia}^2\ ; \qquad i = 1, \ldots, n_c \tag{8}$$

$$c_{ia} \geqslant 0\ ; \qquad i = 1, \ldots, n_c \tag{9}$$

where μ_i is the coefficient of friction acting at the i^{th} contact point. Inequality (8) may be written in matrix form as follows

$$\mathbf{c}_i^{\mathrm{T}}\ \mathbf{D}_i\ \mathbf{c}_i \geqslant 0\ ; \qquad i = 1, \ldots, n_c \tag{10}$$

where

$$\mathbf{D}_i = \frac{1}{1 + \mu_i^2} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & \mu_i^2 \end{bmatrix}$$

Application of the minimum power principle requires that P_{zc} be minimized subject to inequalities (7), (9) and (10). One might think that the equilibrium equation (5) should also be used to constrain the minimization. However, by formulating and examining the dual optimization problem, one finds that equilibrium equation (5) is implicitly satisfied and that inequality (9) is redundant. Thus the primal problem is given by

$$\text{Minimize} \quad P_{zc} = -\dot{\mathbf{q}}_{ob}^{\mathrm{T}} \{\mathbf{g}_{ext} + \begin{bmatrix}\mathbf{W}_n & \mathbf{W}_o\end{bmatrix} \begin{bmatrix}\mathbf{c}_n \\ \mathbf{c}_o\end{bmatrix}\} \qquad (6)$$

$$\text{Subject to:} \quad \mathbf{W}_a^{\mathrm{T}} \dot{\mathbf{q}}_{ob} \geqslant \mathbf{L}_a \dot{\theta} \qquad (7)$$

$$\mathbf{c}_i^{\mathrm{T}} \mathbf{D}_i \mathbf{c}_i \geqslant 0 ; \quad i = 1, \ldots, n_c \qquad (10)$$

with unknowns $\dot{\mathbf{q}}_{ob}$, $\mathbf{c}_n$, $\mathbf{c}_o$ and $\mathbf{c}_a$ (implicit in the vectors $\mathbf{c}_i$).

Applying the Kuhn-Tucker optimality conditions [1] to the primal problem yields the dual constraints. The dual objective function is derived by substituting the velocity constraints into the primal objective function and considering the implications of maximizing the result. After some manipulation, the dual problem is seen to be

$$\text{Maximize} \quad P_c = \dot{\theta}^{\mathrm{T}} \mathbf{L}_a \lambda \qquad (12)$$

$$\text{Subject to:} \quad \mathbf{g}_{ext} + \begin{bmatrix}\mathbf{W}_n & \mathbf{W}_o\end{bmatrix} \begin{bmatrix}\mathbf{c}_n \\ \mathbf{c}_o\end{bmatrix} + \mathbf{W}_a \lambda = 0 \qquad (13)$$

$$\mathbf{W}_n^{\mathrm{T}} \dot{\mathbf{q}}_{ob} + 2\,\mathbf{N}^{\mathrm{T}} \mathbf{D}_n \mathbf{c}_n = 0 \qquad (14)$$

$$\mathbf{W}_o^{\mathrm{T}} \dot{\mathbf{q}}_{ob} + 2\,\mathbf{N}^{\mathrm{T}} \mathbf{D}_o \mathbf{c}_o = 0 \qquad (15)$$

$$2\,\mathbf{N}^{\mathrm{T}} \mathbf{D}_a \mathbf{c}_a = 0 \qquad (16)$$

$$\lambda \geqslant 0 \qquad (17)$$

$$\eta \geqslant 0 \qquad (18)$$

where λ is the vector of Lagrange multipliers associated with inequality (7), $\mathbf{N}$ is a diagonal matrix whose nonzero elements are the Lagrange multipliers η_i associated with inequalities (10), $\mathbf{D}_n$ and $\mathbf{D}_o$ are identity matrices and $\mathbf{D}_a$ is a diagonal matrix with nonzero elements given by $-\mu_i^2$. It is well known that the value of the Lagrange multiplier associated with a specific rigid contact constraint is equal to the magnitude of the normal force necessary to maintain contact [10]. Therefore, the vector λ is equivalent to $\mathbf{c}_a$, and it is evident that constraint (13) is equivalent to the equilibrium equation (5) and constraint (17) is equivalent to inequality (9).

At the optimal solution, the primal and dual constraints are satisfied simultaneously, therefore the primal problem defined by the nonlinear program, (6), (7) and (10), need not include the equilibrium equation. Also, for all feasible solutions, the primal and dual objective functions satisfy the following relationship

$$\dot{\theta}^{\mathrm{T}} \mathbf{L}_a^{\mathrm{T}} \mathbf{c}_a \leqslant -\dot{\mathbf{q}}_{ob}^{\mathrm{T}} \mathbf{g}_{ext} - \dot{\mathbf{q}}_{ob}^{\mathrm{T}} \begin{bmatrix}\mathbf{W}_n & \mathbf{W}_o\end{bmatrix} \begin{bmatrix}\mathbf{c}_n \\ \mathbf{c}_o\end{bmatrix} \qquad (19)$$

with equality holding only at the optimal solution. The term on the left hand side of the inequality is the power applied to the object by the forces of constraint. The terms on the right are the rate of gain of potential energy and the power dissipation through Coulomb friction. Thus, at the optimal solution, expression (19) has the following physical interpretation. The motion of the fingers in the direction of the contact normals supplies power to the object. Some of that power is lost to friction. What remains goes into lifting the object. Consequently every suboptimal solution must defy conservation of energy.

2.1. Extension: Sliding Contacts

The primal forward object motion problem (6), (7) and (10) is complete for rolling contacts, but not for sliding contacts. If sliding occurs at the i^{th} contact, then the i^{th} inequality in (7) and inequality (10) must be satisfied as equalities. The former implies the sliding condition and the latter requires the contact force to lie on the boundary of the friction cone. The Coulomb model of friction also specifies that the contact force be anti-parallel to the relative contact velocity. This specification was concisely expressed by Jameson [7] as

$$\mathbf{c}_i \cdot ({}^i\mathbf{v}_{ti} \times \hat{a}_i) = 0 ; \quad i = 1, \ldots, n_c \qquad (20)$$

$$\mathbf{c}_i \cdot {}^i\mathbf{v}_{ti} \leqslant 0 ; \quad i = 1, \ldots, n_c \qquad (21)$$

where ${}^i\mathbf{v}_{ti}$ is the relative contact velocity (or simply contact velocity) expressed with respect to the i^{th} contact frame. Constraint (20) requires that contact force to lie in the plane formed by the sliding velocity vector and the contact normal. Constraint (21) implies that the friction force oppose the contact velocity, thereby dissipating power. The contact velocity is given by

$${}^i\mathbf{v}_{ti} = \mathbf{W}_i^{\mathrm{T}} \dot{\mathbf{q}}_{ob} - \mathbf{L}_i \dot{\theta} ; \quad i = 1, \ldots, n_c , \qquad (22)$$

where $\mathbf{L}_i$ is the transmitted Jacobian of the i^{th} contact [17]. Relations (20) and (21) may be written in terms of the wrench intensities and the object and arm velocities as the following set of nonsmooth, nonconvex constraints

$$\mathbf{c}_i^{\mathrm{T}} \mathbf{A}_i \mathbf{W}_i^{\mathrm{T}} \dot{\mathbf{q}}_{ob} - \mathbf{c}_i^{\mathrm{T}} \mathbf{A}_i \mathbf{L}_i \dot{\theta} = 0 ; \qquad (23)$$

$$\mathbf{c}_i^{\mathrm{T}} \mathbf{W}_i^{\mathrm{T}} \dot{\mathbf{q}}_{ob} - \mathbf{c}_i^{\mathrm{T}} \mathbf{L}_i \dot{\theta} \leqslant 0 ; \qquad (24)$$

$$i = 1, \ldots, n_c$$

where $\mathbf{A}_i$ is the skew-symmetric cross product matrix associated with the unit normal at the i^{th} contact $\hat{\mathbf{a}}_i$.

The complete primal problem is now given by

$$\text{Minimize} \quad P_{zc} = -\dot{\mathbf{q}}_{ob}^{\mathrm{T}} \{\mathbf{g}_{ext} + \begin{bmatrix}\mathbf{W}_n & \mathbf{W}_o\end{bmatrix} \begin{bmatrix}\mathbf{c}_n \\ \mathbf{c}_o\end{bmatrix}\} \qquad (6)$$

$$\text{Subject to:} \quad \mathbf{W}_a^{\mathrm{T}} \dot{\mathbf{q}}_{ob} \geqslant \mathbf{L}_a \dot{\theta} \qquad (7)$$

$$\mathbf{c}_i^{\mathrm{T}} \mathbf{D}_i \mathbf{c}_i \geqslant 0 ; \quad i \in \Omega \qquad (10)$$

$$\mathbf{c}_i^{\mathrm{T}} \mathbf{A}_i \mathbf{W}_i^{\mathrm{T}} \dot{\mathbf{q}}_{ob} - \mathbf{c}_i^{\mathrm{T}} \mathbf{A}_i \mathbf{L}_i \dot{\theta} = 0 ; \quad i \in \Psi \qquad (23)$$

$$\mathbf{c}_i^{\mathrm{T}} \mathbf{W}_i^{\mathrm{T}} \dot{\mathbf{q}}_{ob} - \mathbf{c}_i^{\mathrm{T}} \mathbf{L}_i \dot{\theta} \leqslant 0 ; \quad i \in \Psi \qquad (24)$$

where Ω and Ψ represent the set of contact points assumed to be maintained and sliding, respectively. More precisely, we write

$$\Omega = \{ i \mid \mathbf{w}_{ai}^{\mathrm{T}} \dot{\mathbf{q}}_{ob} = \mathbf{l}_{ai}^{\mathrm{T}} \dot{\theta} \}$$

$$\Psi = \{ i \mid i \in \Omega \cap \mathbf{c}_i^{\mathrm{T}} \mathbf{D}_i \mathbf{c}_i = 0 \}$$

where $\mathbf{w}_{ai}^{\mathrm{T}}$ is the i^{th} row of $\mathbf{W}_a^{\mathrm{T}}$ and $\mathbf{l}_{ai}^{\mathrm{T}}$ is the i^{th} row of $\mathbf{L}_a$. Constraints (23) and (24) complete the Coulomb friction model without which friction forces could create rather than dissipate power resulting in an unbounded objective function.

To determine $\dot{\mathbf{q}}_{ob}$, P_{zc} must be minimized subject to the rigid body velocity constraints (7) and the Coulomb friction constraints (10), (23) and (24). The object will execute the motion corresponding to the feasible solution of least power. If no feasible solution exists, then the proposed motion of the robot is kinematically inadmissible, *i.e.*, the mechanism will jam. If the minimum power solution is unbounded, then the proposed motion causes the grasp configuration to become unstable, *i.e.*, danger of dropping the object is imminent.

2.2. Special Case: Frictionless Contacts

Before beginning this section, it was claimed that the set of nonlinear programs representing the forward object motion problem reduces to a single linear program when friction is absent. This may be seen by noting that the only nonlinear constraints, (10), (23) and (24), are removed because they are a result of the Coulomb friction model. In addition, since friction forces no longer dissipate power, the second term in the primal object function (and the associated variables $\mathbf{c}_n$ and $\mathbf{c}_o$) becomes zero. Thus in the frictionless case the primal forward object motion problem reduces to the following single linear program, called the *velocity formulation* of the frictionless object motion problem

$$\text{Minimize} \quad P_{xc} = -\dot{\mathbf{q}}_{ob}{}^{T}\, \mathbf{g}_{ext} \tag{25}$$

$$\text{Subject to:} \quad \mathbf{W}_a{}^{T}\, \dot{\mathbf{q}}_{ob} \geqslant \mathbf{L}_a\, \dot{\theta}\ . \tag{7}$$

The input of this formulation is, as before, the vector of joint velocities of the robot. Its output is the contact forces (*i.e.* the Lagrange multipliers associated with inequality (7)), the velocity of the object and the nature of the contact interactions. The contact interactions are indicated by the values of the Lagrange multipliers: if the i^{th} multiplier is zero, then the bodies are separating at the i^{th} contact; if the i^{th} multiplier is positive, then the bodies are sliding on one another at the i^{th} contact; negative values of the multipliers are impossible.

The dual linear program is called the *force formulation* of the frictionless object motion problem and is stated as follows

$$\text{Maximize} \quad P_c = \dot{\theta}^{\,T}\, \mathbf{L}_a\, \mathbf{c}_a \tag{12}$$

$$\text{Subject to:} \quad \mathbf{g}_{ext} + \mathbf{W}_a \mathbf{c}_a = 0 \tag{26}$$

$$\mathbf{c}_a \geqslant 0\ . \tag{27}$$

The velocity and force formulations are equivalent and therefore the input-output relationship for the dual is identical to that of the primal. As was the case for the formulations with friction, the primal and dual solutions are equivalent at the optimal solution so that energy is conserved.

3. Conclusion

The *forward object motion problem* has been developed to predict the instantaneous velocity of an object undergoing quasi-static dexterous manipulation. This formulation is more capable than any previous description of quasi-static manipulation, because any contact configuration is allowed and Coulomb friction constraints are included. The forward problem is in the form of a nonconvex, nonsmooth nonlinear program with all nonlinear terms quadratic or bilinear. Its input is the robot's joint velocities and the contact configuration; its output is the velocity of the object, the nature of contact interactions (*i.e.*, rolling, sliding or separating) and the contact forces. Dramatic simplification occurs in the frictionless case: the nonlinear program reduces to a single linear program. Even if friction is not negligible, if the manipulation task requires specific contact interactions, the nonlinear program may be significantly simplified.

Of the three primary drawbacks of the forward object motion problem two are consequences of applying Peshkin's minimum power principle in the derivation: the first drawback is that only the Coulomb model of friction may be used and the second is that dynamic information is lost. The third drawback is that conservation of energy implies that only the global minimum of the forward object motion problem will predict the object's motion: a local minimum is not good enough. A rectifiable deficiency of the formulation is that statically indeterminate grasp configurations cause difficulty in the numerical solution, because the contact forces cannot be uniquely determined. A remedy which does not violate the conditions of the minimum power principle is to extend the formulation to include compliance.

Application of the forward object motion problem is not restricted to dexterous manipulation. The formulation given in this paper describes the quasi-static motion of a "free" rigid body in contact with a system of velocity-controlled rigid bodies. A straight forward extension of the forward object motion problem could be used to predict the instantaneous velocity of a quasi-static system of "free" rigid bodies in response to the motion of a system of moving rigid boundaries.

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